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Effects of parallel electric field, ion finite Larmor radius and plasma diamagnetic drifts on Alfvén continuum

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Abstract

Based on the drift MHD model, we investigate kinetic effects on the Alfvén continuum, including the finite parallel perturbed electric field ($\delta E_{\parallel}$), ion finite Larmor radius (FLR), and plasma diamagnetic drifts. The $\delta E_{\parallel}$ arises from electron diamagnetic drift and electron Landau damping. It is shown that electron Landau damping couples the kinetic Alfvén wave and the electron drift wave. The electron diamagnetic drift has a much weaker effect (below 1%) compared to the ion diamagnetic drift, as the latter can enhance the frequency of the ion mode up to 20%. We find that ion FLR increases the continuum frequency by more than 20% in the core region, whereas near the edge the enhancement drops to less than 1%. These kinetic effects are also studied for the toroidal Alfvén eigenmode (TAE) within the self-consistent model. While $\delta E_{\parallel}$ has negligible impact on the mode structure and real frequency of TAE (relative frequency shift), it introduces radiative damping with $\gamma/\omega_r \sim 10^{-3}$ through electron Landau damping. These findings emphasize the importance of including kinetic effects in the analysis of Alfvén continuum.

1. Introduction

In burning plasmas, Alfvénic instabilities driven by energetic particles (EPs) constitute a major research topic, as they can significantly degrade EP confinement and thereby affect overall plasma performance [1–3]. Among these instabilities, the toroidal Alfvén eigenmode (TAE) has been extensively studied. The TAE is excited when the drive from wave–particle resonances with EPs exceeds various damping mechanisms [4, 5]. In the ideal MHD limit, a TAE possesses a purely real frequency located within a gap of the shear Alfvén continuous spectrum [6]. If the mode structure of the TAE intersects the continuum, the mode undergoes phase mixing and experiences continuum damping [7, 8]. Kinetic effects introduce additional damping channels, such as Landau damping and radiative damping. When a TAE intersects the continuum, its long-wavelength structure can convert into short-wavelength kinetic Alfvén wave (KAWs), leading to radiative damping [9–11]. Even in the absence of direct intersection, a TAE may still suffer from radiative damping via tunneling effects [12]. For reactor-scale devices like ITER, an accurate calculation is important for accurate Alfvén eigenmode stability assessment, which is critical for predicting EP confinement in future burning plasma experiments [13, 14].

The ideal MHD Alfvén continuum consists of the shear Alfvén wave (SAW) and ion sound wave (ISW) branches, which are coupled through magnetic geodesic curvature. A key feature of this continuum is its gap structure, generated by various mechanisms such as toroidicity and finite- β compression [2, 3, 15]. Discrete eigenmodes reside within these gaps. Starting from the ideal MHD model, one can derive eigenmode equations, and the Alfvén continuum is obtained by setting the second-order radial derivative to zero, as employed in codes such as CONT [6, 15] and ALCON [16]. These tools have been widely used to analyze Alfvén eigenmodes in gyrokinetic simulations [16, 17]. An alternative method for computing the continuum is implemented in the code FALCON [18–20], which solves the MHD equations in ballooning space. This approach is mathematically equivalent to Floquet theory

for ordinary differential equations. While the final results are consistent with those from CONT and ALCON, FALCON additionally evaluates the polarization of the continuum through a parameter termed Alfvénicity. This parameter is particularly important at low frequencies, where coupling between the SAW and ISW branches becomes significant, for example in the continuum of beta-induced Alfvén-acoustic eigenmodes (BAAEs).

The ideal MHD model has inherent limitations, as it omits several important physical effects. In [21], diamagnetic drift was self-consistently incorporated into the continuum calculation, demonstrating its strong influence: ion and EP diamagnetic drift raise the continuum frequency of ion modes and lower that of electron modes. Beyond diamagnetic effects, fully kinetic descriptions of the continuum have also been developed. The code LIGKA employs a gyrokinetic model, based on the quasi-neutrality and gyrokinetic vorticity equations, to compute the kinetic Alfvén continuum [22–24]. This approach extends the continuum calculation to the complex plane, with the damping rate arising from Landau damping of background species, thereby enabling local stability analysis of Alfvénic modes. Another kinetic continuum is calculated by the code DAEPS, which also implements the gyrokinetic model but in ballooning space [25, 26]. However, previous studies based on the gyrokinetic model have mainly focused on the lower frequency range [24, 26], where coupling with ISW is strong. Less attention has been paid to kinetic effects on the Alfvén frequency range, especially the range relevant for TAE.

To illustrate the kinetic effects on the Alfvén continuum within the TAE frequency range, we adopt the drift MHD model, which can faithfully capture essential kinetic effects and provide clear physical insight. Based on equilibrium profiles of DIII-D plasma and ITER baseline scenario, we investigate the influence of $\delta E_{\parallel}$, which arises from electron diamagnetic drift and electron Landau damping. The effects of ion finite Larmor radius (FLR) and ion diamagnetic drift are also taken into account. The results indicate that the continuum frequency is primarily affected by ion FLR and ion diamagnetic drift, whereas the influence of $\delta E_{\parallel}$ and electron diamagnetic drift is considerably weaker. Furthermore, we use the self-consistent theoretical model to calculate the TAE, and the results are consistent with those for the continuum. The $\delta E_{\parallel}$ has almost no effect on the real frequency of TAE, while causing the TAE to experience radiative damping from electron Landau damping. This paper is organized as follows. In section 2, we describe the drift MHD model within the theoretical framework of MAS. The equations for normal modes are discussed in section 2.2. Simulation results for the continuum and eigenmode are presented in section 3. Conclusions are given in section 4. Finally, appendix shows the polarization of the continuum in the ideal MHD model.

2. Physical model

2.1. Drift MHD equations

Within the MAS code, the Landau fluid model accounts for kinetic effects, including the finite parallel perturbed electric field, Landau damping of background particles, FLR effects, and plasma diamagnetic drifts. The full formulations can be found in [27]. Due to the geodesic curvatures in a compressible plasma, the Alfvén wave couples with the sound wave, and the method using polarization to distinguish them is given in appendix. As reported in the first experimental measurement of drift-Alfvén wave polarization on DIII-D, the waves above the geodesic acoustic mode frequency exhibit dominant electromagnetic polarization [28]. The electrostatic component contributed by the ISW branch is negligibly small, indicating that the slow sound approximation can accurately describe the SAW branch in the vicinity of the Alfvén frequency. Therefore, to concentrate on how kinetic effects influence the KAW branch, we adopt this approximation to decouple them while retaining finite- β effects. This leads to the drift MHD model described by the following equations:

$$\frac{\partial}{\partial t} \frac{c}{V_A^2} \nabla_{\perp}^2 \delta \phi + \underbrace{\frac{\partial}{\partial t} (0.75 \rho_i^2 \nabla_{\perp}^2)}_{\{\text{Ion-FLR}\}} \frac{c}{V_A^2} \nabla_{\perp}^2 \delta \phi + i \omega_{*p,i} \frac{c}{V_A^2} \nabla_{\perp}^2 \delta \phi$$

$$+ \mathbf{B}_0 \cdot \nabla \left(\frac{1}{B_0} \nabla_{\perp}^2 \delta A_{\parallel} \right) - \frac{4\pi}{c} \delta \mathbf{B} \cdot \nabla \left(\frac{J_{\parallel 0}}{B_0} \right) - 8\pi \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} \cdot \nabla \delta P = 0, \quad (1)$$

$$\frac{\partial}{\partial t} \delta A_{\parallel} = -c \mathbf{b}_0 \cdot \nabla \delta \phi + \underbrace{\frac{c T_{10^0}}{e n_{10^0}} \mathbf{b}_0 \cdot \nabla \delta n_e + \frac{c T_{10^0}}{e n_{10^0} B_0} \delta \mathbf{B} \cdot \nabla n_{10^0}}_{\{\text{Electron-Drift}\}} + \underbrace{\frac{c m_e}{e} \sqrt{\frac{\pi}{2}} v_{the} |k_{\parallel}| \delta u_{\parallel e}}_{\{\text{Electron-Landau}\}}, \quad (2)$$

$$\frac{\partial}{\partial t} \delta P + \frac{c \mathbf{b}_0 \times \nabla \delta \phi}{B_0} \cdot \nabla P_0 + 2 \Gamma P_0 c \nabla \delta \phi \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} = 0, \quad (3)$$

$$\frac{\partial}{\partial t} \delta n_e + \frac{c \mathbf{b}_0 \times \nabla \delta \phi}{B_0} \cdot \nabla n_{10^0} = 0, \quad (4)$$

where c , e and $m_{\alpha=i,e}$ are the speed of light, particle charge, and mass respectively, $\rho_i = v_{thi}/\Omega_{ci}$, $\Omega_{ci} = Z_i B_0/(cm_i)$, $v_{thi} = \sqrt{T_{i0}/m_i}$, $V_A = B_0/\sqrt{4\pi n_{i0} m_i}$, $v_{the} = \sqrt{T_{10^0}/m_e}$, $k_{||} = -i \mathbf{b}_0 \cdot \nabla$, and $\omega_{*p,i} = \omega_{*n,i} + \omega_{*T,i}$ is the ion diamagnetic drift frequency ($\omega_{*n,i} = -i \frac{c T_{i0}}{Z_i B_0} \mathbf{b}_0 \times \frac{\nabla n_{i0}}{n_{i0}} \cdot \nabla$ and $\omega_{*T,i} = -i \frac{c}{Z_i B_0} \mathbf{b}_0 \times \nabla T_{i0} \cdot \nabla$). $\delta \phi$ is the perturbed electrostatic potential and $\delta A_{||}$ is the parallel perturbed vector potential. $J_{||0} = \frac{c}{4\pi} \mathbf{b}_0 \cdot \nabla \times \mathbf{B}_0$ is the parallel equilibrium current density, $\kappa = \mathbf{b}_0 \cdot \nabla \mathbf{b}_0$ is the magnetic field curvature, and $\delta \mathbf{B} \approx \nabla_{\perp} \times (\delta A_{||} \mathbf{b}_0)$. δn_e is the perturbed electron density, $\delta P = \delta P_e + \delta P_i$ is the total perturbed pressure, and the total equilibrium pressure satisfies $\Gamma P_0 = \Gamma_i P_{i0} + \Gamma_e P_{10^0}$ with $\Gamma_i = \frac{7}{4}$, $\Gamma_e = 1$. The parallel perturbed electron velocity $\delta u_{||e}$ in the {Electron-Landau} term of equation (2) is calculated by inverting parallel Ampère's law:

$$en_{10^0} \delta u_{||e} = \frac{c}{4\pi} \nabla_{\perp}^2 \delta A_{||}. \quad (5)$$

Equations (1)–(5) constitute a closed system that preserves the essential physics of the Landau fluid model. They are helpful for analyzing the effect of $\delta E_{||} = -\mathbf{b}_0 \cdot \nabla \delta \phi - \frac{1}{c} \frac{\partial}{\partial t} \delta A_{||}$, which originates from electron diamagnetic drift and electron Landau damping. Notably, the mechanism of electron diamagnetic drift differs from that of the ion species. Since the plasma current is primarily carried by electrons, the electron diamagnetic drift enters the system through equation (2) (the parallel Ohm's law), whereas the ion diamagnetic drift in equation (1) (vorticity equation) can be regarded as the lowest-order ion FLR effects. An isothermal condition is adopted for electrons in equation (2), with $\mathbf{B} \cdot \nabla T_e = \mathbf{B}_0 \cdot \nabla \delta T_e + \delta \mathbf{B} \cdot \nabla T_{10^0} = 0$. Moreover, it should be noted that the electron diamagnetic drift consists of two terms, marked as {Electron-Drift}, which arise from the linearization of $\mathbf{B} \cdot \nabla n_e = \mathbf{B}_0 \cdot \nabla \delta n_e + \delta \mathbf{B} \cdot \nabla n_{10^0}$. These two terms must be retained together as a physical contribution. In addition, we also retain the ion FLR effects, which is important for the KAW.

2.2. Normal modes

Expanding the drift MHD equations in magnetic flux coordinates (ψ, θ, φ) , where the equilibrium magnetic field is expressed as $\mathbf{B}_0 = q \nabla \psi \times \nabla \theta - \nabla \psi \times \nabla \varphi = I \nabla \theta + g \nabla \varphi$ and the Jacobian is $J^{-1} = B_0^2/(qg + I)$, the governing equation for the Alfvén continuum can be derived by taking the second-order partial derivative with respect to ψ , yielding:

$$\left\{ \begin{aligned} & \frac{\omega^2}{V_A^2} J g^{\psi\psi} - \underbrace{0.75 k_{\perp}^2 \rho_i^2 \frac{\omega^2}{V_A^2} J g^{\psi\psi}}_{\{\text{Ion-FLR}\}} - \underbrace{\frac{\omega \omega_{*p,i}}{V_A^2} J g^{\psi\psi}}_{\{\text{Ion-Drift}\}} - \frac{\omega_{geod}^2}{V_A^2} \frac{2 \kappa_{\theta}^2}{J B_0^2} g^2 R_0^2 \\ & + J \mathbf{B}_0 \cdot \nabla \left(\frac{g^{\psi\psi}}{B_0^2} \mathbf{B}_0 \cdot \nabla \right) - J \mathbf{B}_0 \cdot \nabla \left[\underbrace{\frac{g^{\psi\psi}}{B_0^2} \frac{i \sqrt{\frac{\pi}{2}} \frac{|k_{||}| V_A^2}{v_{the}} k_{\perp}^2 \rho_s^2}}_{\{\text{Electron-Drift}\}} \underbrace{\frac{B_0 \cdot \nabla}{\omega - \omega_{*n,e} + i \sqrt{\frac{\pi}{2}} \frac{|k_{||}| V_A^2}{v_{the}} k_{\perp}^2 \rho_s^2}}_{\{\text{Electron-Landau}\}} \right] \end{aligned} \right\} \frac{\partial^2}{\partial \psi^2} \delta \phi = 0, \quad (6)$$

where $\omega_{*n,e} = i \frac{c T_{10^0}}{e B_0} \mathbf{b}_0 \times \frac{\nabla n_{10^0}}{n_{10^0}} \cdot \nabla$ is the electron density diamagnetic drift frequency, $\rho_s = C_s/\Omega_{ci}$, $C_s = \sqrt{T_{10^0}/m_i}$, $\kappa_{\theta} = \frac{g q}{J B_0^3} \frac{\partial B_0}{\partial \theta}$, $\omega_{geod}^2 = 2 \Gamma P_0/(n_{i0} m_i R_0^2)$, and $g^{\psi\psi} = |\nabla \psi|^2$, and we assume $Z_i = e$ and $Z_i n_{i0} = en_{10^0}$. We apply transformation $\partial_t \rightarrow -i\omega$ and retain the $k_{\perp} = -i \nabla_{\perp}$ in {Ion-FLR} and {Electron-Landau}. For continuum calculation, we approximate $k_{\perp} \approx \sqrt{2} k_{\theta}$. Equation (6) is consistent with equation (5) in [21]. Furthermore, we incorporate the ion FLR effects, electron diamagnetic drift, and electron Landau damping. By omitting the terms labeled {Ion-FLR}, {Ion-Drift}, {Electron-Landau} and {Electron-Drift}, the equation reduces to that of the Alfvén continuum in the ideal MHD model under the slow sound approximation.

Considering cylindrical geometry and setting the second-order partial derivative to zero, we obtain an approximate dispersion relation for the normal modes:

$$\left(1 - \frac{\omega_{*n,e}}{\omega} \right) \left[\frac{\omega_{SAW}^2}{\omega^2 (1 - 0.75 k_{\perp}^2 \rho_i^2) - \omega \omega_{*p,i}} - 1 \right] = i \sqrt{\pi} |\xi_e| \frac{\omega_{SAW}^2}{\omega^2} k_{\perp}^2 \rho_s^2, \quad (7)$$

Table 1. Descriptions of different reduced models and approximate frequency for the ion mode. ω_{MHD} is the approximate frequency in the ideal MHD model, and $\gamma_{\text{ELD}}^{\text{IV}}$, $\gamma_{\text{ELD}}^{\text{V}}$ represent the electron Landau damping rate.

Model	ion diamagnetic drift	ion FLR	electron diamagnetic drift	electron Landau damping	Approximate frequency
{I}	×	×	×	×	ω_{MHD}
{II}	✓	×	×	×	$\omega_{\text{MHD}} + \frac{1}{2}\omega_{*p,i}$
{III}	✓	✓	×	×	$\omega_{\text{MHD}} \sqrt{1 + 0.75k_{\perp}^2 \rho_i^2} + \frac{1}{2}\omega_{*p,i}$
{IV}	×	×	✓	✓	$\omega_{\text{MHD}} + i\gamma_{\text{ELD}}^{\text{IV}}$
{V}	×	✓	✓	✓	$\omega_{\text{MHD}} \sqrt{1 + 0.75k_{\perp}^2 \rho_i^2} + i\gamma_{\text{ELD}}^{\text{V}}$

where $|\xi_e| = \omega / (\sqrt{2}|k_{\parallel}|v_{the})$, and $\omega_{\text{SAW}} = k_{\parallel} V_A$ is the SAW frequency in the incompressible ideal MHD model. The {Electron-Landau} term introduces coupling between the electron drift wave (EDW) and the KAW. The former satisfies $\omega = \omega_{*n,e}$, while the latter satisfies $\omega \approx \pm \omega_{\text{SAW}} \sqrt{1 + 0.75k_{\perp}^2 \rho_i^2} + \frac{1}{2}\omega_{*p,i}$. For parameters satisfying $|\xi_e| < 1$ in the Alfvén frequency range and $k_{\perp} \rho_s < 1$, the EDW and KAW remain nearly decoupled. This coupled dispersion relation is physically analogous to equation (15) in the recent MAS drift-tearing mode study [29]. Both formulations realize the coupling effects via a dissipative term. The electron Landau damping introduced by the finite parallel electric field in this work corresponds to the resistive dissipation term in that reference.

All numerical calculations in this work are performed within the framework of the MAS eigenvalue code, a well-benchmarked Landau-fluid code for plasma stability analysis in general toroidal geometry [27]. For numerical implementation, the linearized drift MHD equations are discretized in magnetic flux coordinates and cast into a standard generalized eigenvalue problem $\mathbb{A}X = \omega \mathbb{B}X$, where $\mathbb{A}$ and $\mathbb{B}$ are sparse radial operator matrices that are independent of the eigenvalue ω . For the Alfvén continuum calculation, only the coefficients of the second-order radial derivative are retained in the operators, consistent with the conventional continuum definition. For the full TAE eigenmode calculation, all operator terms are preserved to capture the complete radial mode structure. Detailed numerical schemes and benchmark results of the MAS code can be found in [21, 27].

3. Simulation results

In this section, we simulate the Alfvén continuum to investigate the influence of kinetic effects. To obtain a clear physical understanding, particularly of the ion FLR effects and $\delta E_{\parallel}$, the analysis is organized around five distinct models, summarized in table 1. The table also lists the corresponding approximate frequencies for each model, and Model-{I} represents the ideal MHD model. Simulations are performed using two different equilibrium profiles, corresponding to DIII-D plasma (L-mode) and the ITER baseline scenario (H-mode), respectively.

3.1. DIII-D plasma

We first use the equilibrium profile of DIII-D shot #142 111 [21, 30], as shown in figure 1. The background plasma parameters on axis are $n_{ea} = 2.99 \times 10^{19} \text{ m}^{-3}$, $T_{ea} = 2049 \text{ eV}$, $\beta_{ea} = 0.66\%$, and $B_a = 1.94 \text{ T}$. From the core ($\rho_{\text{tor}} = 0.1$) to the edge ($\rho_{\text{tor}} = 0.9$), the ρ_i decreases by up to 70% and the ρ_s decreases by up to 80%. In the DIII-D experiment, the $n = 3, 4, 5$ TAEs are found to be unstable and located at the $q = 4.5$ rational surface. Figure 2 compares the frequencies of the $n = 4$ continua for the ion mode obtained from the different models.

The difference between Model {I} and {II} arises from the inclusion of $\omega_{*p,i}$. In figure 2(a), it can be seen that $\omega_{*p,i}$ introduces a relative frequency upshift of up to 20%, especially in the region $\rho_{\text{tor}} < 0.4$. This phenomenon is consistent with the approximate frequency in table 1 and the discussion in [21]. Physically, the frequency upshift originates from the ion diamagnetic term in the equation (1), which acts as the lowest-order FLR correction and introduces a real frequency component proportional to the radial gradient of ion pressure. As the minor radius increases, the pressure gradient decreases but safety factor increases, leading to a monotonically weakening diamagnetic effect toward the plasma edge, in good agreement with the scaling $\omega_{*p,i} \propto \nabla P_i / q$.

Figure 2(b) presents the continuum frequency computed for Model {III}. A direct comparison with Model {I} reveals that ion FLR effects raise the continuum frequency in the core region ($\rho_{\text{tor}} < 0.2$) by more than 20%. This upshift can be understood analytically from the approximate frequency for Model {III} in table 1, which consistent with KAW dispersion. Beyond the $k_{\theta} \propto r^{-1}$ scaling that reduces $k_{\perp} \rho_i$

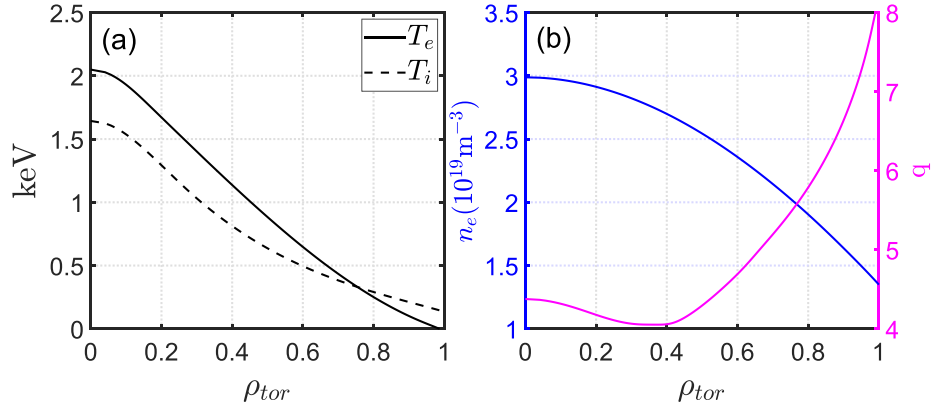


Figure 1. Plasma radial profiles in DIII-D shot #142 111: (a) ion and electron temperature T_α ($\alpha = i, e$), (b) electron density n_e and safety factor q . $\rho_{tor} = \sqrt{\psi_T/\psi_{Tw}}$ is the square root of the normalized toroidal magnetic flux.

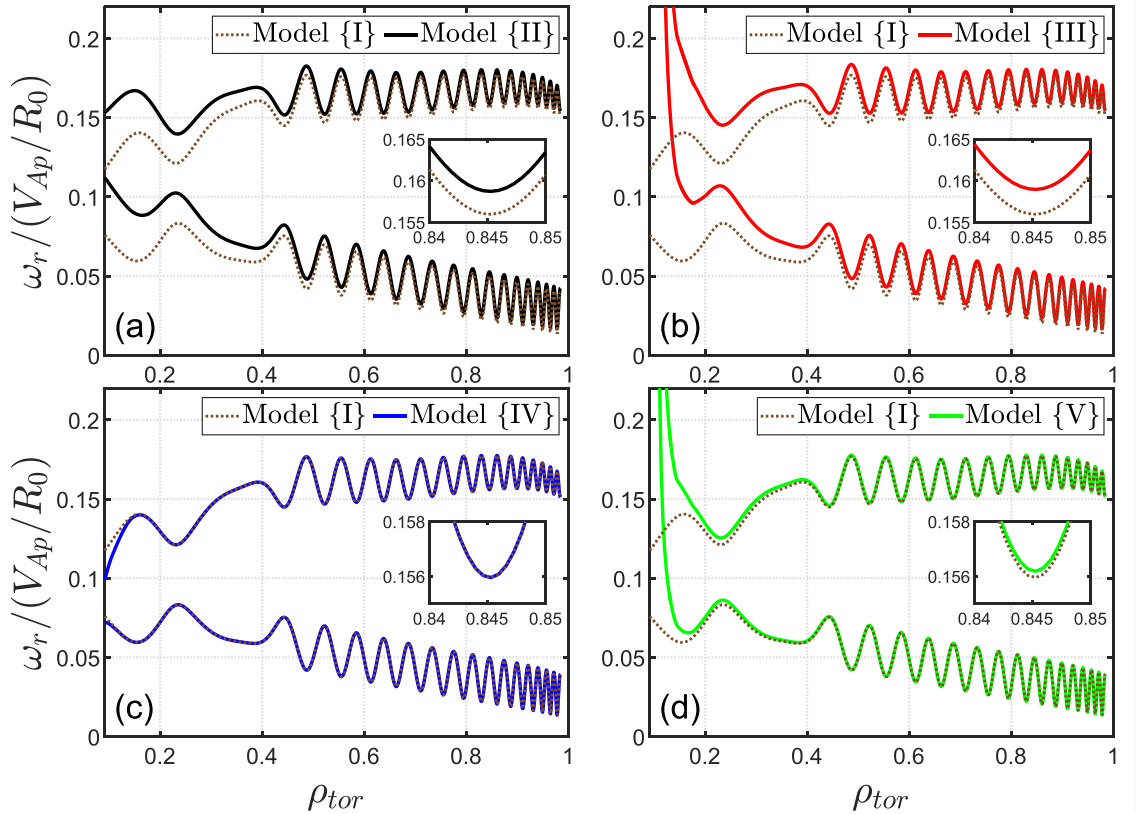


Figure 2. The $n = 4$ Alfvén continua calculated by different models. $V_{Ap} = B_a / \sqrt{4\pi n_e m_p}$, and descriptions of the models are given in table 1.

toward the edge, the radial decay of FLR effects is further amplified by the decreasing ion temperature profile shown in figure 1(a). Since $\rho_i \propto \sqrt{T_i}$, the reduction of ρ_i at larger radii suppresses the FLR correction, forming a dual dependence on k_θ and T_i .

Next, we illustrate the impact of $\delta E_\parallel$ on the Alfvén continuum, which arises from $\omega_{*n,e}$ and electron Landau damping. A comparison between figures 2(a) and (c) shows that the effect of $\omega_{*n,e}$ is considerably weaker than that of $\omega_{*p,i}$, with a relative frequency shift below 1% across the entire radial profile. Two underlying mechanisms account for this weak influence. First, under the isothermal electron assumption adopted in this work, $|\omega_{*p,e}| = |\omega_{*n,e}| \ll |\omega_{*p,i}| < |\omega_{MHD}|$ due to the absence of electron temperature gradient contribution. Second, as indicated by the equation (7), the EDW branch nearly decouples from the KAW branch due to $|\xi_e| = \omega / (\sqrt{2}|k_\parallel|v_{the}) < 1$ in the Alfvén frequency range and $k_\theta \rho_s < 1$ away from the core region. These results indicate that the kinetic coupling strength between drift waves and Alfvén continua exhibits strong frequency dependence, and the weak coupling assumption is well

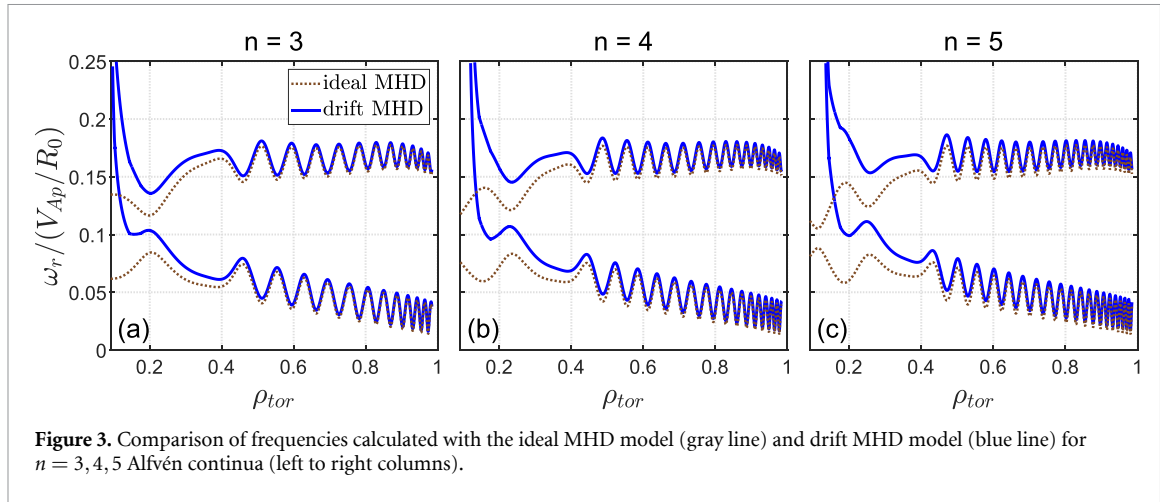


Figure 3. Comparison of frequencies calculated with the ideal MHD model (gray line) and drift MHD model (blue line) for $n = 3, 4, 5$ Alfvén continua (left to right columns).

justified for the TAE-relevant Alfvén frequency band. However, in the low-frequency regime, gyrokinetic simulations reveal enhanced coupling between drift waves and Alfvén-acoustic branches with increasing $T_{e,i}$ in the BAAE frequency range [24, 26], while in the drift-tearing mode regime, $\omega_{*n,e}$ above a critical threshold drives strong mode coupling and substantially modifies both mode growth rate and radial structure [29]. Model {V} further includes the ion FLR effects, and the frequency is shown in figure 2(d). The phenomenon is consistent with that in figure 2(b), namely that ion FLR increases the frequency as the minor radius decreases.

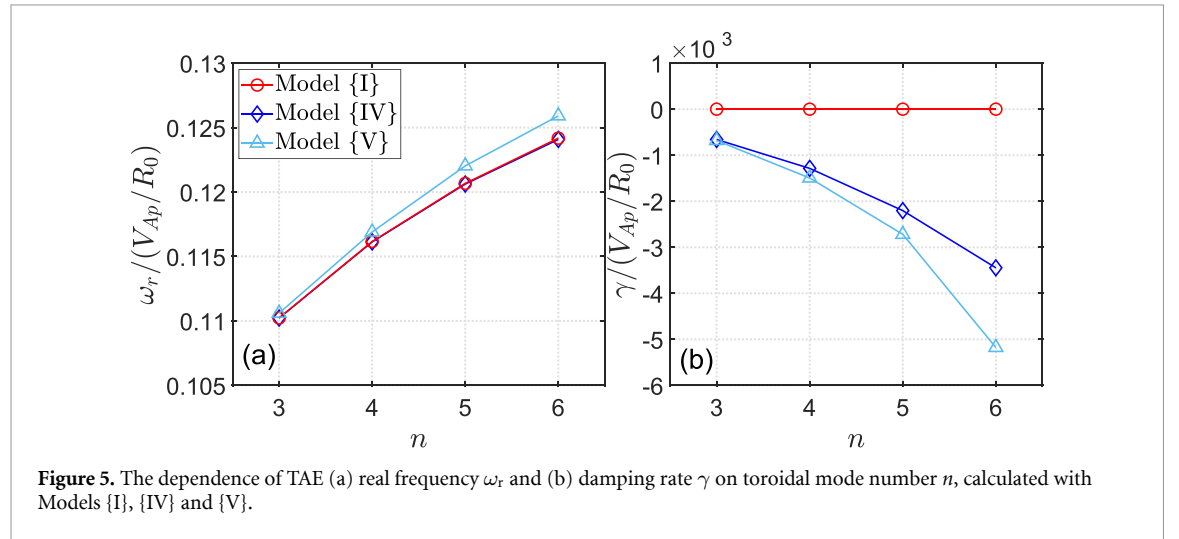
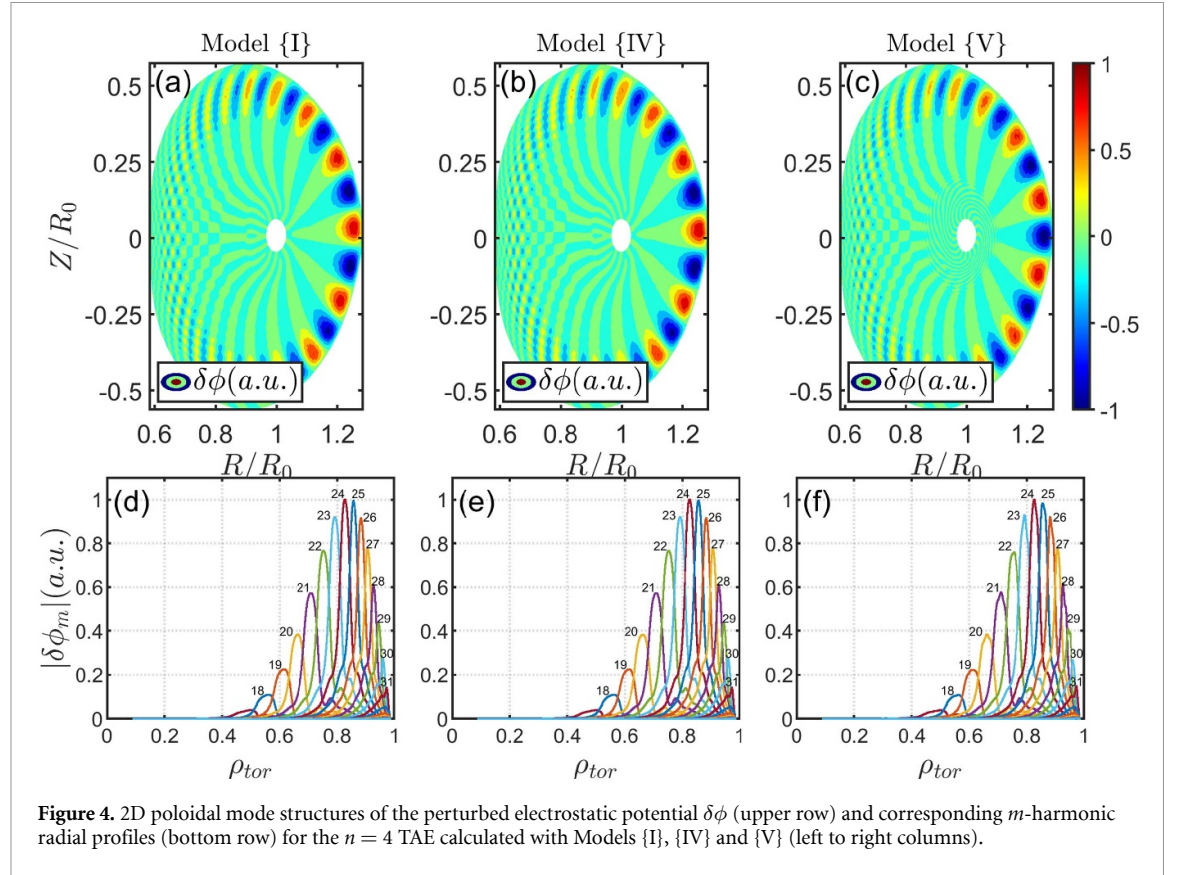
Considering the scalings $\omega_{*p,i} \propto n$ and $k_\theta \propto m \propto nq$, the effects of both $\omega_{*p,i}$ and ion FLR are expected to strengthen with increasing toroidal mode number n . Based on these scalings, we examine the continua for $n = 3, 4, 5$, as shown in figure 3. The results reveal that the enhancement of the continuum frequency in the core region ($\rho_{\text{tor}} < 0.4$) becomes progressively more pronounced as n increases. This observation suggests that $\omega_{*p,i}$ and ion FLR effects are important and should be considered for high- n modes. In short summary, we find that $\omega_{*p,i}$ and ion FLR contribute significantly to raising the continuum frequency for the ion mode. In contrast, the continuum frequency remains nearly unaffected by $\delta E_\parallel$.

Figure 4 shows the mode structure of the $n = 4$ TAE calculated with Models {I}, {IV}, and {V}. To minimize continuum damping, all analyzed TAEs are localized in the outer radial region, away from intersections with the continuum. Given that $k_\perp \rho_i < 1$ and $k_\perp \rho_s < 1$, the mode structure itself is nearly unaffected by ion FLR effects and $\delta E_\parallel$. Figure 5 summarizes the real frequencies ω_r and damping rates γ of TAEs from these models. It is evident that ω_r values from Models {I} and {IV} are nearly identical, indicating that $\delta E_\parallel$ has negligible effects on ω_r . Moreover, since $\delta E_\parallel$ involves electron Landau damping and is related to radiative damping, a small damping rate of the TAE appears in Model {IV}. Although the TAE does not intersect the continuum, it still suffers from radiative damping via tunneling effects [12, 27]. However, as expected from the condition $|v_{the}/R_0| \gg |\omega_{\text{TAE}}|$, the damping rate induced by $\delta E_\parallel$ is very small. Figure 5 also shows ω_r and γ obtained from Model {V}. Small quantitative differences emerge among them. Due to $k_\theta \propto nq$, the dependence on n becomes more sensitive. Consequently, the TAE in this equilibrium is only weakly damped, with kinetic effects manifesting as a real frequency shift rather than a strong damping channel, consistent with the continuum results. It is worth mentioning that the theoretical models describing both the continuum and the eigenmode are self-consistent, both based on the drift MHD model. This consistent framework facilitates a unified physical understanding of kinetic effects, whether they impact the eigenmode directly or indirectly through the continuum.

3.2. ITER baseline scenario

The equilibrium profile of the ITER baseline scenario [13] is shown in figure 6. The background plasma parameters on axis are $n_{ea} = 1.13 \times 10^{20} \text{ m}^{-3}$, $T_{ea} = 26.96 \text{ keV}$, $\beta_{ea} = 4.43\%$ and $B_a = 5.3 \text{ T}$. From the edge pedestal top ($\rho_{\text{tor}} = 0.9$) to the core ($\rho_{\text{tor}} = 0.1$), the ρ_i decreases by up to 65% and ρ_s decreases by up to 70%. Differing from the equilibrium profile of DIII-D shot #142 111, there is an edge pedestal in the ITER baseline scenario. The temperatures and densities of electron and ion change significantly near the edge ($\rho_{\text{tor}} > 0.95$).

The $n = 20$ continua for the ion mode obtained from different models are presented in figure 7. The continuum structure of Model {I} differs from that shown in figure 2. In the ITER baseline scenario, the central frequency of the TAE gap declines from approximately 0.4 (V_{Ap}/R_0) in the core to about



0.1 (V_{Ap}/R_0) near the edge. Since this frequency scales roughly as $V_A/(2qR_0)$, the observed decrease can be attributed to the combined effects of a flat density profile and an increasing q profile in the region $0.4 < \rho_{tor} < 0.9$. This radially decreasing continuum structure influences the radial localization of global TAEs, which is consistent with previous gyrokinetic analysis for the ITER baseline scenario [23].

More importantly, the kinetic effects on the continuum are qualitatively consistent with observations in DIII-D plasmas. Specifically, the frequency is upshifted by the $\omega_{*p,i}$ and ion FLR effects, while $\delta E_{||}$ has little impact. In the core region ($\rho_{tor} < 0.4$), the relative frequency shift from $\omega_{*p,i}$ is only 10%, significantly smaller than those in DIII-D. This difference arises because the much higher magnetic field and density in ITER yield a larger ω_{MHD} , so the ratio $|\omega_{*p,i}/\omega_{MHD}|$ is reduced. This scaling confirms that the relative importance of diamagnetic corrections depends on the ratio of drift frequency to Alfvén frequency, which is generally lower in larger, higher-field reactor-scale devices. In contrast, for $\rho_{tor} > 0.95$, $\omega_{*p,i}$ significantly increases the frequency, primarily due to the steep ion temperature and density gradients in the edge pedestal, which is a feature absent in the DIII-D L-mode equilibrium.

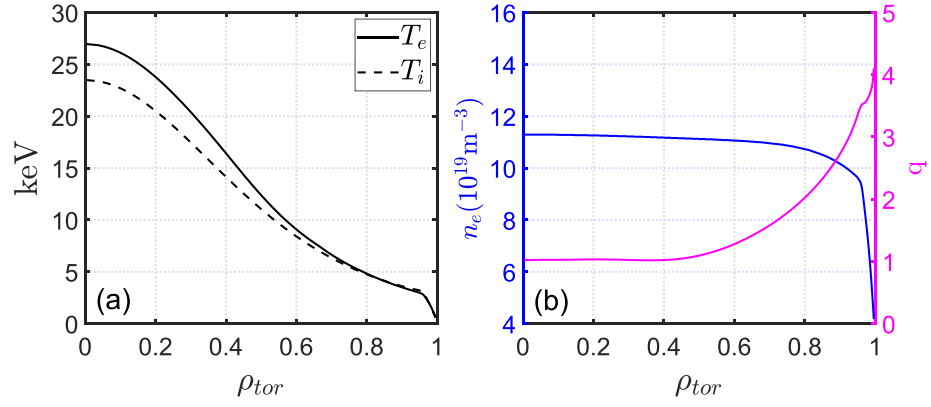


Figure 6. Plasma radial profiles in the ITER baseline scenario: (a) ion and electron temperature T_α ($\alpha = i, e$), (b) electron density n_e and safety factor q .

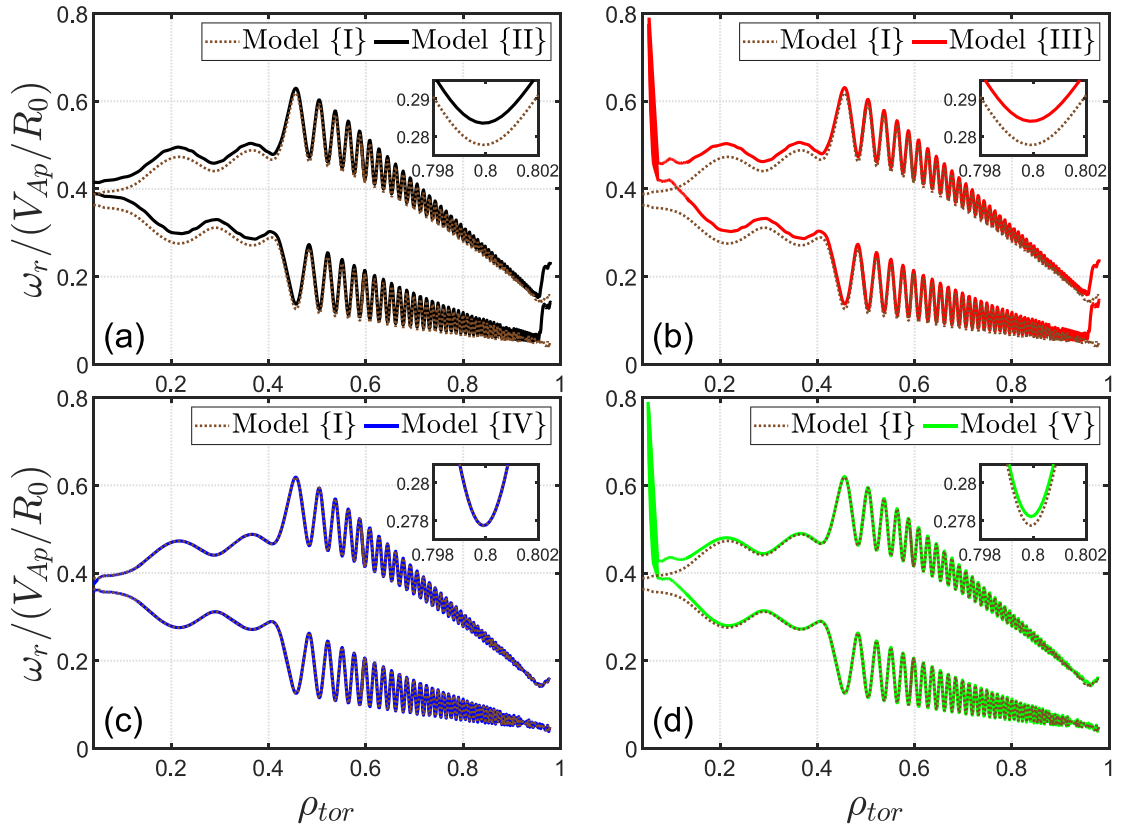


Figure 7. The $n = 20$ Alfvén continua calculated by different models. Descriptions of the models are given in table 1.

Considering the optimal α -particle drive (the drift orbit width of α -particle matches the mode width), the toroidal mode numbers of interest for Alfvén eigenmodes in the ITER baseline scenario are around $n \sim 30$ [23], which are higher than those in DIII-D plasmas. This increase is attributed to the larger device size, which reduces the stabilizing influence of FLR and finite orbit width (FOW) effects on the α -particle drive for high- n modes. Figure 8 presents the continua for toroidal mode numbers $n = 20, 30, 40$. Compared to figure 3, the frequency upshift is more pronounced, which can be attributed to ion FLR in the core and $\omega_{*p,i}$ near the edge. Furthermore, due to the broader range of n considered, the differences among the continua become more substantial. These findings underscore the necessity of including ion FLR and $\omega_{*p,i}$ effects in analyses of Alfvén physics for the ITER baseline scenario.

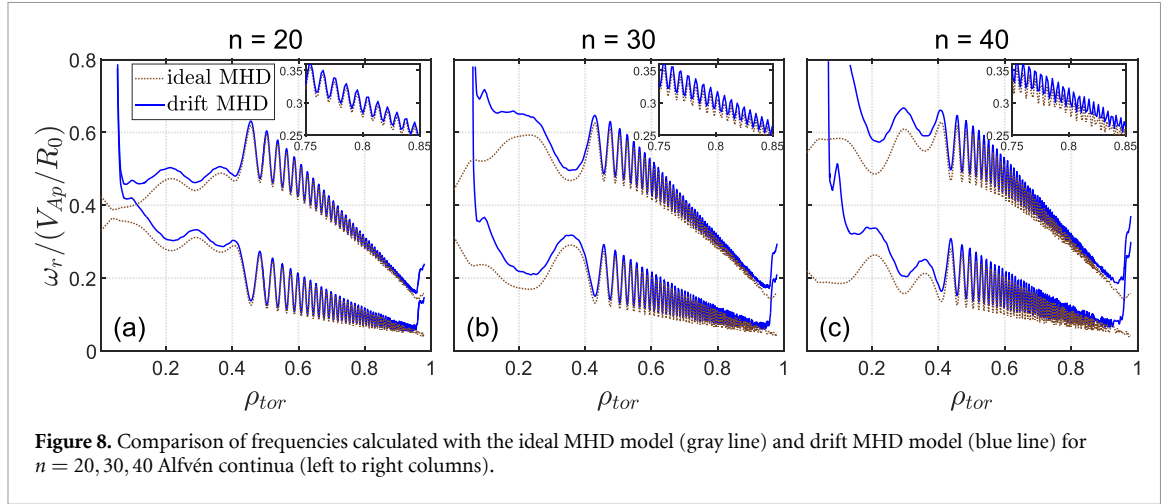


Figure 8. Comparison of frequencies calculated with the ideal MHD model (gray line) and drift MHD model (blue line) for $n = 20, 30, 40$ Alfvén continua (left to right columns).

4. Conclusions

In this work, we have investigated the effects of $\delta E_{\parallel}$, ion FLR, and plasma diamagnetic drifts on the Alfvén continuum and eigenmode within the framework of the MAS theoretical model. To obtain a clear and self-consistent physical picture, both the continuum and eigenmode are described by the drift MHD model, incorporating the slow sound approximation to decouple the ISW branch while retaining essential kinetic effects. The main results can be summarized as follows:

- (1) The underlying mechanisms associated with $\omega_{*p,e}$ and $\omega_{*p,i}$ are fundamentally different. Specifically, $\omega_{*p,i}$ appears in the vorticity equation as a result of the lowest-order ion FLR effects. In contrast, $\omega_{*p,e}$ arises in the parallel Ohm's law because the plasma current is primarily carried by electrons. Under the assumption of isothermal electrons, we obtain $\omega_{*p,e} = \omega_{*n,e}$.
- (2) The $\omega_{*p,i}$ enhances the continuum frequency for the ion mode. In the core region of DIII-D plasma, this enhancement reaches up to 25%. In the core of ITER baseline scenario, however, the enhancement is weaker, owing to $|\omega_{*p,i}| \ll |\omega_{\text{MHD}}|$. In contrast, due to the steep gradients of ion temperature and density in the edge pedestal, the continuum frequency near the edge region of the ITER baseline scenario is significantly increased by $\omega_{*p,i}$.
- (3) The contribution of $\delta E_{\parallel}$, which arises from electron diamagnetic drift and electron Landau damping, has a negligible effect on the continuum frequencies. Within the Alfvén frequency range and for $k_{\perp} \rho_s < 1$, the EDW and KAW remain nearly decoupled. Moreover, because $|\omega_{*n,e}| \ll |\omega_{\text{MHD}}|$, the electron diamagnetic drift exerts a much weaker influence than ion diamagnetic drift, leaving the continuum frequency nearly identical to that in the ideal MHD model. These trends are also observed for TAEs, where $\delta E_{\parallel}$ contributes only weak radiative damping.
- (4) The ion FLR effects elevate both continuum and TAE frequencies, particularly in the core plasma, while its influence diminishes toward the edge. Furthermore, ion FLR effects increase the sensitivity of both the frequency and damping rate of TAEs to the toroidal mode number.

These results are generally consistent with the studies based on both ideal-MHD with diamagnetic corrections [21] and gyrokinetic models [23, 24, 26], but extend the current understanding in several aspects. Compared with [21], which considered diamagnetic drift effects of ions, this work further identifies the dominant role of ion FLR and ion diamagnetic drift in the Alfvén frequency range. Compared with gyrokinetic continuum studies that focused on the low-frequency acoustic-coupling regime [24, 26], our work clarifies that electron drift-kinetic coupling becomes significantly weaker in the TAE frequency band, which justifies the use of reduced drift-MHD models for efficient TAE stability analysis. In future DT experiments, the inclusion of multiple ion species will be critical for accurate eigenmode stability analysis and related physical investigations. Extending the present kinetic continuum framework to account for multiple particle species thus represents an important direction for future research, which we intend to pursue in the next phase of this work.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Appendix. Polarization of Alfvén continuum in ideal MHD model

The continuum in the ideal MHD model within the MAS code has been well benchmarked, and polarization is used to distinguish the SAW and ISW branches. Introducing $\delta\phi = -\frac{1}{c}\frac{\partial U}{\partial t}$, the governing equations for the continuum can be written as

$$\frac{\omega^2}{V_A^2} J g^{\psi\psi} \frac{\partial^2 U}{\partial \psi^2} = -J \mathbf{B}_0 \cdot \nabla \left(\frac{g^{\psi\psi}}{B_0^2} \mathbf{B}_0 \cdot \nabla \right) \frac{\partial^2 U}{\partial \psi^2} + 8\pi \Gamma P_0 J \frac{\kappa_\theta}{q} \frac{\partial}{\partial \psi} \left(\frac{\delta P}{\Gamma P_0} \right), \quad (8)$$

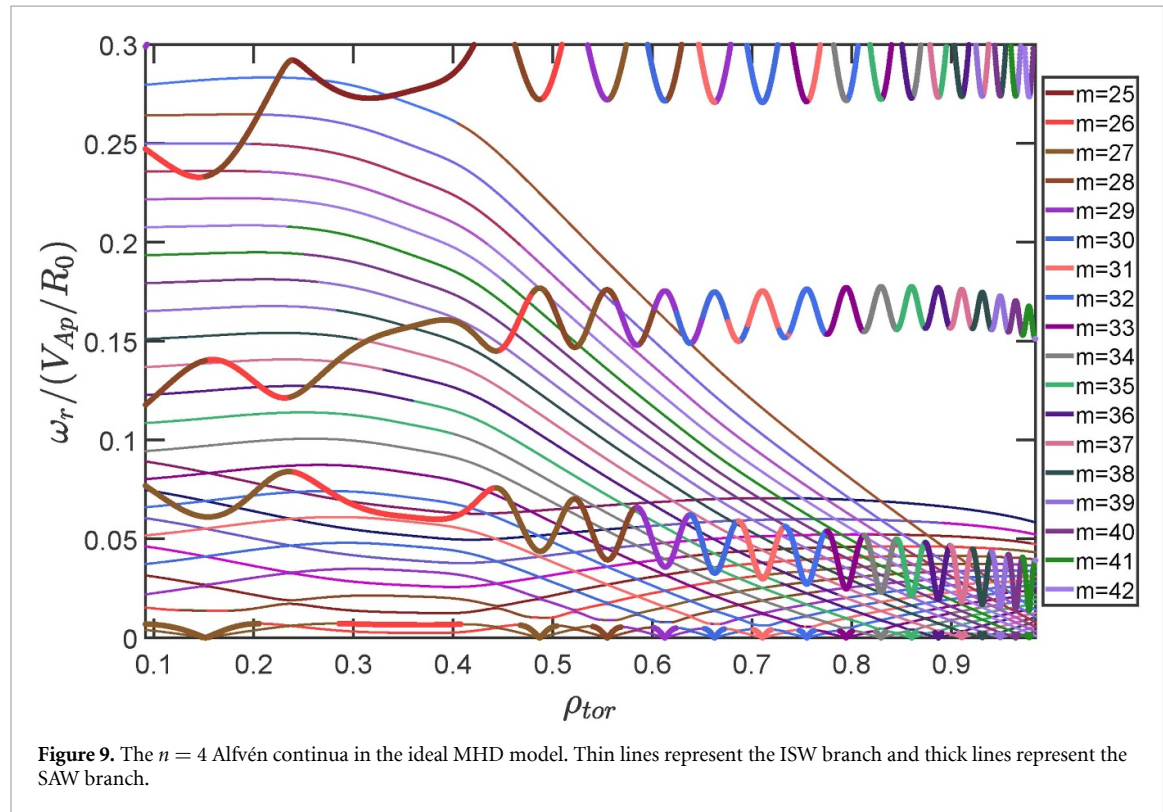
$$\omega^2 J \frac{\partial}{\partial \psi} \left(\frac{\delta P}{\Gamma P_0} \right) \underbrace{- 2\omega^2 J \frac{\kappa_\theta}{q} \frac{\partial^2 U}{\partial \psi^2}}_{\{\text{geodesic}\}} = \underbrace{- C_s^2 J \mathbf{B}_0 \cdot \nabla \left(\frac{1}{J B_0^2} \mathbf{B}_0 \cdot \nabla \right) \frac{\partial}{\partial \psi} \left(\frac{\delta P}{\Gamma P_0} \right)}_{\{\text{sound wave}\}}, \quad (9)$$

where $C_s = \sqrt{\Gamma P_0 / (n_{i0} m_i)}$ is the sound speed. It is evident that the geodesic curvatures in the second term on the right-hand side of equation (8) and the second term on the left-hand side of equation (9) result in coupling between the SAW and ISW branches. For the Alfvén frequency range with the slow sound approximation ($\omega^2 \gg k_\parallel^2 C_s^2$), the {sound wave} term can be removed to decouple the SAW and ISW branches. Therefore, we can use the {geodesic} and {sound wave} terms in equation (9) to define

the polarization:

$$P = \frac{|\{\text{geodesic}\}|^2}{|\{\text{geodesic}\}|^2 + |\{\text{sound wave}\}|^2}. \quad (10)$$

If the polarization P exceeds 0.5, i.e. $|\{\text{geodesic}\}| > |\{\text{sound wave}\}|$, it is identified as the SAW branch. The calculation results with the equilibrium profile of DIII-D plasma are shown in figure 9, where the SAW and ISW branches are well separated, consistent with results from NOVA, ALCON, and FALCON. Moreover, the BAAE continuum, resulting from coupling between SAW and ISW branches, can also be distinguished in the low frequency range $\omega_r < 0.01$ (V_{Ap}/R_0).



References

- [1] Fasoli A *et al* 2007 Chapter 5: Physics of energetic ions *Nucl. Fusion* **47** S264
- [2] Heidbrink W W 2008 Basic physics of Alfvén instabilities driven by energetic particles in toroidally confined plasmas *Phys. Plasmas* **15** 055501
- [3] Chen L and Zonca F 2016 Physics of Alfvén waves and energetic particles in burning plasmas *Rev. Mod. Phys.* **88** 015008
- [4] Fu G Y and Van Dam J W 1989 Excitation of the toroidicity-induced shear Alfvén eigenmode by fusion alpha particles in an ignited tokamak *Phys. Fluids B* **1** 1949
- [5] Fu G Y, Cheng C Z, Budny R, Chang Z, Darrow D S, Fredrickson E, Mazzucato E, Nazikian R and Zweben S 1995 Stability Analysis of toroidicity-induced Alfvén eigenmodes in TFTR deuterium-tritium experiments *Phys. Rev. Lett.* **75** 2336
- [6] Cheng C Z and Chance M S 1986 Low- n shear Alfvén spectra in axisymmetric toroidal plasmas *Phys. Fluids* **29** 3695
- [7] Zonca F and Chen L 1992 Resonant damping of toroidicity-induced shear-Alfvén eigenmodes in tokamaks *Phys. Rev. Lett.* **68** 592
- [8] Spong D A, Carreras B A and Hedrick C L 1992 Linearized gyrofluid model of the alpha-destabilized toroidal Alfvén eigenmode with continuum damping effects *Phys. Fluids B* **4** 3316–28
- [9] Mett R R and Mahajan S M 1992 Kinetic theory of toroidicity-induced Alfvén eigenmodes *Phys. Fluids B* **4** 2885–93
- [10] Fu G Y, Berk H L and Pletzer A 2005 Kinetic damping of toroidal Alfvén eigenmodes *Phys. Plasmas* **12** 082505
- [11] Lauber P, Günter S and Pinches S D 2005 Kinetic properties of shear Alfvén eigenmodes in tokamak plasmas *Phys. Plasmas* **12** 122501
- [12] Berk H L, Mett R R and Lindberg D M 1993 Arbitrary mode number boundary-layer theory for nonideal toroidal Alfvén modes *Phys. Fluids B* **5** 3969–96
- [13] Polevoi A R, Medvedev S Y, Mukhovatov V S, Kukushkin A S, Murakami Y, Shimada M and Ivanov A A 2002 ITER confinement and stability modelling *J. Plasma Fusion Res. Ser.* **5** 82–87 (available at: https://www.jspf.or.jp/JPFERS/PDF/Vol5/jpfrs2002_05-082.pdf)
- [14] Qiu Z *et al* 2025 Alpha particle confinement in CFEDR H-mode scenario *Plasma Sci. Technol.* **27** 104005

- [15] Chu M S, Greene J M, Lao L L, Turnbull A D and Chance M S 1992 A numerical study of the high- n shear Alfvén spectrum gap and the high- N gap mode *Phys. Fluids B* **4** 3713
- [16] Deng W, Lin Z, Holod I, Wang Z, Xiao Y and Zhang H 2012 Linear properties of reversed shear Alfvén eigenmodes in the DIII-D tokamak *Nucl. Fusion* **52** 043006
- [17] Cheng J Y, Zhang W L, Lin Z, Holod I, Li D, Chen Y and Cao J 2016 Gyrokinetic particle simulation of fast-electron driven beta-induced Alfvén eigenmode *Phys. Plasmas* **23** 052504
- [18] Falessi M V, Carlevaro N, Fusco V, Vlad G and Zonca F 2019 Shear Alfvén and acoustic continuum in general axisymmetric toroidal geometry *Phys. Plasmas* **26** 082502
- [19] Falessi M V, Carlevaro N, Fusco V, Giovannozzi E, Lauber P, Vlad G and Zonca F 2020 On the polarization of shear Alfvén and acoustic continuous spectra in toroidal plasmas *J. Plasma Phys.* **86** 845860501
- [20] Pucella G *et al* 2020 Beta-induced Alfvén eigenmodes and geodesic acoustic modes in the presence of strong tearing activity during the current ramp-down on JET *Plasma Phys. Control. Fusion* **64** 045023
- [21] Bao J, Zhang W L, Li D and Lin Z 2020 Effects of plasma diamagnetic drift on Alfvén continua and discrete eigenmodes in tokamaks *J. Fusion Energy* **39** 382–9
- [22] Lauber P *et al* 2009 Kinetic Alfvén eigenmodes at ASDEX Upgrade *Plasma Phys. Control. Fusion* **51** 124009
- [23] Lauber P 2015 Local and global kinetic stability analysis of Alfvén eigenmodes in the 15 MA ITER scenario *Plasma Phys. Control. Fusion* **57** 054011
- [24] Bierwage A and Lauber P 2017 Gyrokinetic analysis of low- n shear Alfvén and ion sound wave spectra in a high-beta tokamak plasma *Nucl. Fusion* **57** 116063
- [25] Li Y, Hu S, Zheng W and Xiao Y 2020 Drift Alfvén energetic particle stability with circulating particles *Phys. Plasmas* **27** 062505
- [26] Li Y, Falessi M V, Lauber P, Li Y, Qiu Z, Wei G and Zonca F 2023 Physics of drift Alfvén instabilities and energetic particles in fusion plasmas *Plasma Phys. Control. Fusion* **65** 084001
- [27] Bao J *et al* 2023 MAS: a versatile Landau-fluid eigenvalue code for plasma stability analysis in general geometry *Nucl. Fusion* **63** 076021
- [28] Du X D, Chen L, Heidbrink W W, Zeeland M A V, Austin M E, Chen J, Liu Y Q, McKee G R and Yan Z 2024 First measurement of drift-Alfvén wave polarization in magnetically confined fusion *Plasmas Phys. Rev. Lett.* **132** 215101
- [29] Chen Y M, Bao J, Cai H S and Wang Z X 2026 Benchmark of MAS kinetic-MHD code for the 2/1 drift-tearing mode in fusion plasmas *Phys. Plasmas* **33** 062502
- [30] Wang Z, Lin Z, Holod I, Heidbrink W W, Tobias B, Van Zeeland M and Austin M E 2013 Radial localization of toroidicity-induced Alfvén eigenmodes *Phys. Rev. Lett.* **111** 145003