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ABSTRACT

The isotope effect on zonal flow is a critical issue for turbulent transport in burning plasmas. In this work, we conduct global simulations of the isotope effects of hydrogen isotopes (H, D, and T) on the zonal flow across arbitrary wavelengths using the Gyrokinetic Toroidal Code. Our results reveal that the residual zonal flow (RZF) level exhibits a significant isotope effect primarily at intermediate wavelengths (i.e., $k_r \rho_i \sim 1$). The strength of this isotope effect and the wavelength at which it is strongest are mainly influenced by isotope mass M_i and safety factor q , with a lesser impact from the inverse aspect ratio ϵ . The isotope effect intensifies as the isotope mass increases from H to T or as the safety factor q increases. Simulations incorporating kinetic electrons show that the contribution of electron polarization to the RZF level is minimal compared to that of thermal ions at intermediate wavelengths, indicating that electron polarization has a negligible effect on the zonal flow isotope effect. Additionally, our simulation results for the isotope effect on RZF, as well as on geodesic acoustic mode frequency and damping rate, align with analytic theories. However, we suggest that earlier analytic theory may overestimate the isotope effect on RZF in this crucial intermediate wavelength range. Overall, our findings indicate that the isotope effect on zonal flow may contribute to isotope turbulent transport, particularly in trapped electron turbulence and turbulence in high- q region.

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I. INTRODUCTION

Anomalous heat transport in tokamaks is primarily attributed to plasma turbulence stemming from drift wave instabilities, notably ion temperature gradient (ITG) mode, and trapped electron mode (TEM). The self-generated zonal flow is recognized as a key regulator of turbulent transport.^{1–4} It consists of a near-zero frequency component and an oscillatory component called geodesic acoustic mode (GAM).^{5,6} In toroidal plasmas, while the latter decays to zero with a well-defined damping rate, the former relaxes toward a nonzero residual value in the absence of collisions,⁷ which is usually referred to as residual zonal flow (RZF).⁷

The onset of burning plasmas in tokamak experiments has unveiled an isotope effect on turbulent transport, particularly evident in electron and edge transport phenomena.^{8,9} A significant aspect contributing to this isotope dependency is the potential reliance of RZF on the mass of isotopes. Consequently, the investigation of the isotope effect on RZF has emerged as a crucial concern in burning plasmas, recently explored through analytical theory,¹⁰ albeit with limited confirmation from gyrokinetic simulations.¹¹ The analytic theory of Hahm suggests that the isotope effect may play an important role in

intermediate wavelength zonal flow,¹⁰ which could mainly affect electron turbulence. It is known that code verification is an important ingredient in gyrokinetic code development. Hence, in addition to its important physics implication, Hahm's theory can be also used effectively to verify gyrokinetic codes that want to study isotope turbulent transport.

In this study, we utilize global gyrokinetic simulations with the Gyrokinetic Toroidal Code (GTC)¹ to investigate the isotope effects on RZF across various wavelengths, with a careful cross-check with earlier theory.¹⁰ First, we establish the intrinsic consistency between the gyrokinetic formulation in the GTC code and the classical theory of Rosenbluth–Hinton (R-H) regarding zonal flow response. Our results demonstrate a quantitative agreement in the trends of residual zonal flow with respect to the radial wavenumber k_r , aligning with the semi-analytic calculation of Xiao–Catto (X-C)¹² and the analytic theory of Wang and Hahm.¹³ Notably, the isotope effect is most pronounced at intermediate wavelengths, i.e., $k_r \rho_i \sim 1$, as found in the theory of Hahm *et al.*¹⁰ However, our findings indicate that this theory may overestimate the isotope effect on the residual zonal flow at these intermediate wavelengths, which is crucial for accurately evaluating electron

transport in TEM turbulence. Additionally, our simulation results are consistent with analytic predictions for GAM frequency and damping rate.⁶ After examining the effects of electron polarization, safety factor q , inverse aspect ratio ε , and ion mass M_i on residual zonal flow, we find that q and M_i are the primary parameters influencing the isotope effect in ion-scale turbulence. By considering the wavelength range and q values that exhibit a strong isotope effect, the results of this paper suggest that isotope transport may play a more significant role in CTEM turbulence and turbulence in high- q region.

The paper is structured as follows: Sec. II reviews the gyrokinetic theory relevant to RZF and outlines the gyrokinetic particle simulation model employed in the GTC code. In Sec. III, we use GTC simulations to investigate the isotope effects on residual zonal flow levels, as well as on GAM frequency and damping rate, and we compare the RZF levels with and without electron polarization. We then examine the influence of important factors, such as q , ε , and M_i , on the isotope effect in RZF. Finally, Sec. IV presents our conclusions and related discussions.

II. GYROKINETIC PARTICLE SIMULATION MODEL

In this section, we review the analytic theory of residual zonal flow and compare it with the formulation used in the GTC simulation.

A. Gyrokinetic formulation for zonal flow response

The zonal flow, or equivalently the axisymmetric electrostatic potential $\langle\phi\rangle = \phi_{n=0,m=0}$, can drive a first-order perturbed distribution function f_1 . Here, the electrostatic potential is expressed as $\phi = \langle\phi\rangle + \delta\phi$ and the total distribution function as $F = f_1 + F_0$. The perturbed distribution f_1 can be decomposed into linear and nonlinear components $f_1 = f_1^L + f_1^{NL}$. It is common to separate the adiabatic response from the distribution by defining $f_1^L = -\frac{\varepsilon}{T}F_0\phi + g e^{-iL_k}$, where $L_k = \frac{k \cdot v_\perp}{\Omega}$. Consequently, the gyrocenter distribution g driven by zonal potential is simply given by

$$\frac{\partial g_k}{\partial t} + (v_\parallel \mathbf{b} \cdot \nabla + i\omega_d)g_k = \frac{e}{T}J_0 F_0 \frac{\partial \langle\phi\rangle_k}{\partial t}, \quad (1)$$

in the Fourier space, where $\omega_d = \mathbf{k}_\perp \cdot \mathbf{v}_d = v_\parallel \mathbf{b} \cdot \nabla Q$, with $Q = \frac{k_\perp v_\parallel}{\Omega_p}$. We then express g_k as $g_k = H_k e^{-iQ}$ and transform the preceding equation from the gyrocenter to the banana center. The banana center distribution H_k satisfies the following equation:

$$\frac{\partial H_k}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla H_k = e^{iQ} \frac{e}{T} J_0 F_0 \frac{\partial \langle\phi\rangle_k}{\partial t}. \quad (2)$$

To the lowest order, the solution to the preceding equation is $H_k = e^{iQ} \frac{\varepsilon}{T} J_0 F_0 \langle\phi\rangle_k$. Consequently, the perturbed function f_k is given by

$$f_k^L = -\frac{e}{T} F_0 \langle\phi\rangle_k (1 - e^{iQ} J_0 e^{-iQ} e^{-iL_k}). \quad (3)$$

This expression is valid for both ions and electrons, encompassing both classical and neoclassical polarization. The quasi-neutrality condition is required to close the system

$$\sum_{s=i,e} \left(1 - \frac{1}{n_0} \int d^3 v F_0 e^{iQ_s} J_{0s}^2 e^{-iQ_s} \right) \frac{4\pi e_s^2 n_0}{T_s} \langle\phi\rangle_k = 4\pi e \delta n^{NL}. \quad (4)$$

This can be written as

$$\langle k_\perp^2 \rangle \epsilon_{pol} \langle\phi\rangle_k = 4\pi e \delta n^{NL}, \quad (5)$$

where the polarization constant is contributed by both ions and electrons in general, i.e., $\epsilon_{pol} = \epsilon_{pol}^i + \epsilon_{pol}^e$. The polarization constant for species s is defined as

$$\epsilon_{pol}^s \equiv \left(1 - \frac{1}{n_0} \int d^3 v F_0 e^{iQ_s} J_{0s}^2 e^{-iQ_s} \right) \frac{4\pi e_s^2 n_0}{T_s k_\perp^2}, \quad (6)$$

where $s = i, e$, and both finite Larmor radius (FLR) effect and finite orbit width (FOW) effect are taken into account. It should be noted that the factor e^{iQ_s} represents the FOW effect. In the zero orbit width limit, i.e., $Q_s = 0$, the polarization becomes

$$\epsilon_{pol}^s = \frac{4\pi e_s^2 n_0}{T_s \langle k_\perp^2 \rangle} (1 - \Gamma_0(k_\perp \rho_s)). \quad (7)$$

Assuming a long wavelength, i.e., $k_\perp \rho_s \ll 1$, we find that $\epsilon_{pol}^s = \frac{\omega_{ps}^2}{\Omega_s^2} = \frac{4\pi n_0 m_s}{B^2} \equiv \epsilon_{pol}^{cl}$, which corresponds to the classical polarization arising from gyromotion or the FLR effect. It is evident that the classical polarization is dominated by ions rather than electrons. In the long-time limit $t \rightarrow \infty$, the polarization constant ϵ_{pol}^s can be flux surface averaged,

$$\epsilon_{pol}^s = \left(1 - \frac{1}{n_0} \left\langle \int d^3 v F_0 e^{iQ_s} J_{0s}^2 e^{-iQ_s} \right\rangle \right) \frac{4\pi e_s^2 n_0}{T_s \langle k_\perp^2 \rangle}, \quad (8)$$

where both classical and neoclassical polarization are coupled together in the same expression. For ion-scale turbulence, such as ion temperature gradient (ITG) mode and trapped electron mode (TEM), the condition $k_\perp \rho_e \ll 1$ generally holds. As a result, the FLR effect can be neglected, and the electron polarization can be described using a drift electron model,

$$\epsilon_{pol}^e \equiv \left(1 - \frac{1}{n_0} \left\langle \int d^3 v F_0 e^{iQ_e} e^{-iQ_e} \right\rangle \right) \frac{4\pi e^2 n_0}{T_e \langle k_\perp^2 \rangle}. \quad (9)$$

Therefore, only the neoclassical polarization is relevant for electrons in ion-scale turbulence.

B. Residual zonal flow

Rosenbluth and Hinton (R-H) developed a theory for residual zonal flow, demonstrating that the effectiveness of the regulation is due to the finite nature of this residual flow. In the original R-H zonal flow model, turbulent fluctuations generate a charge source within a time frame that is much shorter than the bounce time but significantly longer than the gyroperiod. This turbulence-induced charge source produces an initial zonal flow potential through classical polarization, $\epsilon_{pol}^{s,cl}$, which results from particles departing from their gyrocenters. This process occurs over several ion gyration periods. After multiple bounce times, the initial potential is modified by the total polarization, ϵ_{pol}^s , which includes both classical polarization and neoclassical polarization, $\epsilon_{pol}^{s,nc}$, arising from the gyrocenter's departure from the banana center. The total polarization in Eq. (8) can be written as

$$\epsilon_{pol}^s = \epsilon_{pol}^{s,cl} + \epsilon_{pol}^{s,nc} - \frac{1}{n_0} \left\langle \int d^3 v F_0 (1 - J_{0s}^2)(1 - J_{0s}^2) \right\rangle \frac{4\pi e_s^2 n_0}{T_s \langle k_\perp^2 \rangle}, \quad (10)$$

where $\epsilon_{pol}^{s,cl}$ and $\epsilon_{pol}^{s,nc}$ can be expressed as

$$\epsilon_{pol}^{s,cl} = \frac{1}{n_0} \left\langle \int d^3 v F_0 (1 - J_{0s}^2) \right\rangle \frac{4\pi e_s^2 n_0}{T_s \langle k_\perp^2 \rangle},$$

and

$$\epsilon_{pol}^{s,nc} = \frac{1}{n_0} \left\langle \int d^3 v F_0 (1 - J_{Qs}^2) \right\rangle \frac{4\pi e_s^2 n_0}{T_s \langle k_\perp^2 \rangle}, \quad \text{with } J_{Qs} = \overline{e^{iQ_s}},$$

according to Wang and Hahm.¹³ In the long wavelength limit, the last coupling term in Eq. (10) is small on the order of $(k_\perp \rho_s)^4$ and can be neglected. Therefore, the total polarization can be conveniently written as the sum of classical and neoclassical polarization, as is done in W-H 2009.

According to R-H, the residual level of the zonal flow, R_{ZF} , is given by the following expression:

$$R_{ZF} \equiv \frac{\langle \phi \rangle_k(t = \infty)}{\langle \phi \rangle_k(t = 0)} = \frac{\epsilon_{pol}^{i,cl}}{\epsilon_{pol}^i + \epsilon_{pol}^{e,nc}}, \quad (11)$$

which can be further conveniently written as

$$R_{ZF} = \frac{1}{1/R_i + \left(\epsilon_{pol}^{e,nc} / \epsilon_{pol}^{e,cl} \right) \left(\epsilon_{pol}^{e,cl} / \epsilon_{pol}^{i,cl} \right)}, \quad (12)$$

where the polarization constants $\epsilon_{pol}^{i,cl}$, $\epsilon_{pol}^{e,cl}$, ϵ_{pol}^i , and $\epsilon_{pol}^{e,nc}$ are defined in the preceding section, and the ion residual function R_i is defined as

$$R_i \equiv \frac{\epsilon_{pol}^{i,cl}}{\epsilon_{pol}^i} = \frac{1 - \Gamma_0(k_\perp \rho_i)}{1 - \frac{1}{n_0} \left\langle \int d^3 v F_0 e^{iQ_i} J_{0i}^2 e^{-iQ_i} \right\rangle}. \quad (13)$$

The electron residual function R_e can be defined in a similar way. Therefore, the R-H residual zonal flow level can be expressed as

$$R_{ZF} = \frac{R_i}{1 + \frac{R_i}{R_e} \frac{T_i}{T_e} \frac{1 - \Gamma_0(k_\perp \rho_e)}{1 - \Gamma_0(k_\perp \rho_i)}}. \quad (14)$$

C. Simulation model in GTC

In the GTC code, we solve the perturbed gyrokinetic equation for the linear gyroaveraged perturbation $\bar{f}_1^L$ as follows:

$$\begin{aligned} \frac{\partial \bar{f}_k^L}{\partial t} + (v_\parallel \mathbf{b} \cdot \nabla + i\omega_d) \bar{f}_k^L \\ = -\frac{e}{T} J_0 F_0 (v_\parallel \mathbf{b} \cdot \nabla + i\omega_d) \phi_k + \mathbf{v}_E \cdot \nabla F_0. \end{aligned} \quad (15)$$

A corresponding nonlinear form of the preceding linear equation has been implemented in GTC to study nonlinear ion dynamics. As is common in gyrokinetic simulations, we explicitly separate the classical polarization from the perturbation f_1^L to enforce quasineutrality, thereby deriving the gyrokinetic Poisson equation for the electrostatic potential ϕ ,

$$\sum_{s=i,e} \frac{4\pi e_s^2}{T_s} n_0 (\phi - \tilde{\phi}) = 4\pi \sum_{s=i,e} \int d^3 v f_{1s} J_{0s}. \quad (16)$$

In the Fourier space, the preceding equation can be written as

$$\sum_{s=i,e} \frac{4\pi e_s^2}{T_s} n_0 \phi (1 - \Gamma_0(k_\perp \rho_s)) = 4\pi \sum_{s=i,e} \int d^3 v f_{1s} J_{0s}. \quad (17)$$

For the zonal potential response, the linear gyrokinetic equation becomes

$$\frac{\partial \bar{f}_k^L}{\partial t} + (v_\parallel \mathbf{b} \cdot \nabla + i\omega_d) \bar{f}_k^L = -\frac{e}{T} J_0 F_0 i\omega_d \langle \phi \rangle_k. \quad (18)$$

Letting $\bar{f}_k^L = h_k e^{-iQ}$, we find that h_k satisfies

$$\frac{\partial h_k}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla h_k = -\frac{e}{T} J_0 F_0 v_\parallel \mathbf{b} \cdot \nabla (e^{iQ} \langle \phi \rangle_k). \quad (19)$$

By letting $h_k = \bar{H}_k - \frac{e}{T} J_0 F_0 e^{iQ} \langle \phi \rangle_k$, we find that $\bar{H}_k$ satisfies

$$\frac{\partial \bar{H}_k}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla \bar{H}_k = e^{iQ} \frac{e}{T} J_0 F_0 \frac{\partial}{\partial t} \langle \phi \rangle_k. \quad (20)$$

This equation can be solved perturbatively in the long-time limit, $\omega/\omega_b \ll 1$, yielding

$$\bar{H}_k = H_k = \overline{e^{iQ}} \frac{e}{T} J_0 F_0 \langle \phi \rangle_k. \quad (21)$$

Thus, we obtain

$$\bar{f}_k^L = -\frac{e}{T} J_0 F_0 \langle \phi \rangle_k (1 - e^{-iQ} \overline{e^{iQ}}). \quad (22)$$

This expression is very close to the previously derived Eq. (3), though not exactly identical. Substituting this expression into the gyrokinetic Poisson equation Eq. (17) yields

$$\begin{aligned} \sum_{s=i,e} \frac{4\pi e_s^2}{T_s} n_0 (1 - \Gamma_0(k_\perp \rho_s)) \langle \phi \rangle_k + \sum_{s=i,e} \frac{4\pi e_s^2}{T_s} n_0 \\ \times \left(\frac{1}{n_0} \int d^3 v F_0 J_{0s}^2 - \frac{1}{n_0} \int d^3 v F_0 \overline{e^{iQ_s}} J_{0s}^2 e^{-iQ_s} \right) \langle \phi \rangle_k = 4\pi e \delta n^{NL}. \end{aligned} \quad (23)$$

The second term exactly cancels the third term, leading to

$$\sum_{s=i,e} \frac{4\pi e_s^2}{T_s} n_0 \left(1 - \frac{1}{n_0} \int d^3 v F_0 \overline{e^{iQ_s}} J_{0s}^2 e^{-iQ_s} \right) \langle \phi \rangle_k = 4\pi e \delta n^{NL}, \quad (24)$$

which is exactly the same as Eq. (4). Thus, we have demonstrated that the formulation used in the GTC simulation accurately describes the linear response of zonal flow. However, it is important to address a subtle aspect of electron dynamics. In the electron kinetic-fluid hybrid model implemented in the GTC code, the electron drift kinetic equation is solved for the non-adiabatic electron distribution g_e ,¹⁴ where $g_e = f_{1e} - \frac{e}{T_e} F_0 \delta \phi$. The non-adiabatic electron distribution g_e satisfies the following equation:

$$\begin{aligned} \frac{\partial g_e}{\partial t} + (v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_E) \cdot \nabla g_e - \mathbf{b}^* \cdot \nabla (\mu B - e\phi) \frac{1}{m_e} \frac{\partial g_e}{\partial v_\parallel} \\ = -F_0 \left[\frac{\partial}{\partial t} \left(\frac{e\delta\phi}{T_e} \right) + \delta \mathbf{v}_E \cdot \nabla \ln F_0 - (\mathbf{v}_d + \delta \mathbf{v}_E) \cdot \nabla \frac{e\langle \phi \rangle}{T_e} \right]. \end{aligned} \quad (25)$$

For the linear zonal flow response, the perturbed drive $\delta\phi$ on the right-hand side of the preceding equation is neglected, resulting in

$$\frac{\partial g_e}{\partial t} + (v_{\parallel} \mathbf{b} + \mathbf{v}_d) \cdot \nabla g_e - \mathbf{b}^* \cdot \nabla (\mu B - e\phi) \frac{1}{m_e} \frac{\partial g_e}{\partial v_{\parallel}} = F_0 \mathbf{v}_d \cdot \nabla \frac{e\langle\phi\rangle}{T_e}. \quad (26)$$

In the linear limit, this can be equivalently expressed as

$$\frac{\partial g_{ek}}{\partial t} + (v_{\parallel} \mathbf{b} \cdot \nabla + i\omega_d) g_{ek} = \frac{e}{T} J_0 F_0 i\omega_d \langle\phi\rangle_k, \quad (27)$$

which is consistent with Eq. (18). Therefore, the electron drift kinetic formulation in the GTC code can effectively capture the essential zonal flow response for $k_{\perp} \rho_e \ll 1$, which is further supported by the following GTC simulation of the R-H residual zonal flow.

III. SIMULATION RESULTS

To investigate the isotope effect on the residual zonal flow level, we employ three hydrogen isotopes (H, D, and T) as the thermal ions in our simulations. These simulations are carried out using a circular tokamak model characterized by concentric flux surfaces. The radial domain of the simulation is centered on a reference magnetic flux surface with a radius of $r = 0.5a$. At this reference flux surface, key parameters include a safety factor of $q = 1.4$ and an inverse aspect ratio of $\varepsilon = r/R_0 = 0.2$. Additionally, pertinent parameters at the magnetic axis include the magnetic field $B_0 = 2.0\text{T}$, ion and electron temperature $T_i = T_e = 2.2\text{keV}$, $\tau = q_i T_e / (e T_i) = 1.0$, and the electron density $n_e = 7.9 \times 10^{19}/\text{m}^3$. Initially, an external radial potential field $\langle\phi\rangle_{\text{ext}}$ with a specific radial wavelength k_r is introduced, followed by an adjustment of the particle density in accordance with $\langle\phi\rangle_{\text{ext}}$ over some period of time. Subsequently, the simulation time is reset to $t = 0$, and we start to solve zonal flow $\langle\phi\rangle$ and monitor the temporal evolution of the zonal potential. To ensure an adequate representation of radial wavelengths within the simulation domain and to mitigate boundary effects, the major radius is set to $R_0 = 664\text{cm}$, 166cm , and 83.5cm for $k_r \leq 0.2$, $0.2 < k_r \leq 1.0$, and $k_r > 1.0$, respectively.

A. Banana orbit width scaling

Before simulating the zonal flow response, we undertake a validation process to ensure the accurate implementation of isotope scaling for particle bounce motion within the GTC code. This validation involves benchmarking the banana orbit width (Δ_B) of trapped ions against theoretical isotope scaling. Theoretical predictions dictate that the orbit width adheres to $\Delta_B \propto \sqrt{M_i T_i} / (qB)$, with an expected monotonic increase proportional to the square root of the isotope mass, i.e., $\propto \sqrt{M_i}$. Employing the major radius $R_0 = 83.5\text{cm}$, our simulation results agree with this expectation. As shown in Fig. 1(a), the orbit width consistently enlarges when transitioning from hydrogen isotopes H to T. Across isotopes H, D, T, and two other artificial isotopes ^5H and ^7H , Δ_B spans a range from $12\rho_H$ to $38\rho_H$. Here, ^5H and ^7H are included just for numerical verification purpose, as their half-lives ($\sim 10^{-23} - 10^{-21}\text{s}$) are much shorter than the particle transit period in the simulations. Notably, this empirical data aligns quantitatively with analytical predictions, as illustrated in Fig. 1(b). Consequently, our findings affirm that the banana orbit width adheres to the correct isotope scaling within the GTC code.

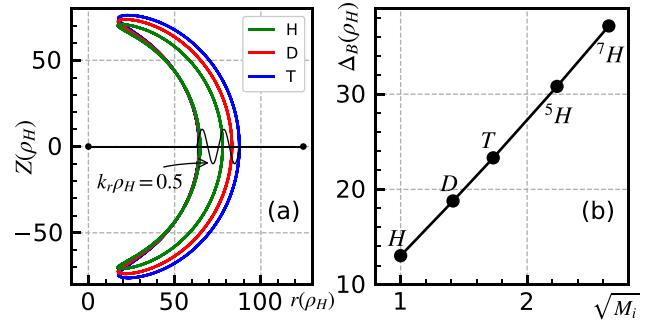


FIG. 1. (a) Typical banana orbit of trapped ions in the poloidal plane for hydrogenic isotopes (H, D, T) with $R_0 = 83.5\text{cm}$, and (b) banana orbit width varies as a function of the square root of ion mass.

B. Boundary treatment and convergence study

As per Eq. (11), the residual zonal flow level R_{ZF} is determined by evaluating $\langle\phi\rangle(t = \infty)$ in the long-time limit through gyrokinetic simulation. It is found that selecting an appropriate boundary treatment of the external potential $\langle\phi\rangle_{\text{ext}}$ is crucial for achieving an accurate residual zonal flow response. Within our simulations, we experimented with three distinct radial boundary treatments, labeled as BT-1, BT-2, and BT-3, respectively.

For the BT-1 boundary treatment, we choose a straightforward approach by initiating a sinusoidal radial function,

$$\langle\phi\rangle_{\text{ext}}(r) = A_{\text{ext}} \sin(k_r(r - 0.5a) + \pi). \quad (28)$$

Here, A_{ext} represents the fixed external potential amplitude, and $k_r = 2\pi N_{\lambda} / \Delta r$, where N_{λ} denotes the number of wavelengths in the radial simulation domain, and Δr represents the domain size. For instance, if $N_{\lambda} = 8$ is set, an 8-wavelength sinusoidal wave is launched in the radial domain. The BT-1 boundary treatment retains all N_{λ} wavelengths within the simulation domain and ensures that $\langle\phi\rangle_{\text{ext}}(r)$ vanishes solely at the two boundaries, while $\langle E \rangle_{\text{ext}}(r) = -d\langle\phi\rangle_{\text{ext}}/dr$ does not necessarily vanish at these boundaries.

The BT-2 boundary treatment adjusts $\langle\phi\rangle_{\text{ext}}(r)$ to approach zero near the boundaries by shifting and scaling it for $\frac{3}{4}$ wavelength near the inner boundary and $\frac{3}{4}$ wavelength near the outer boundary. Consequently, BT-2 maintains $N_{\lambda} - 2$ exact waves for the simulation. Although both $\langle\phi\rangle_{\text{ext}}(r)$ and $\langle E \rangle_{\text{ext}}(r)$ approach zero at the boundaries under BT-2, the second and higher-order derivatives of $\langle\phi\rangle_{\text{ext}}(r)$ are not necessarily continuous.

For the BT-3 boundary treatment, we multiply the following super-Gaussian envelope to the sinusoidal function that is used in the BT-1 boundary treatment:

$$A_{\text{ext}}(r) = \exp \left[- \left(\frac{r - 0.5a}{0.3\Delta r} \right)^6 \right]. \quad (29)$$

Hence, BT-3 allows continuous second and higher-order derivatives for $\phi_{\text{ext}}(r)$ near the two boundaries, albeit at the expense of preserving fewer exact wavelengths compared to BT-2. Here, the factor 0.3 in Eq. (29) is an empirical value. It controls the width of the flat-top of the super-Gaussian envelope, so decreasing the factor leads to fewer exact wavelengths. Setting the factor as 0.3 achieves a balance between the grid number for boundary treatment and the grid number for the exact zonal potential.

In the GTC code, Dirichlet boundary conditions or zero boundary conditions are applied to both zonal flow and zonal density. When particles reach the inner or outer boundaries, they are returned to their previous positions, with the exception that their poloidal positions are adjusted from θ to $2\pi - \theta$. This adjustment preserves momentum in an up-down symmetric tokamak equilibrium. While this treatment ensures that particle trajectories remain within the simulation domain, it effectively truncates trajectories that extend beyond the boundaries. To minimize the impact of this boundary treatment and achieve accurate R_{ZF} , the simulation domain size (Δr) should be significantly larger than the typical banana orbit width (Δ_B).

It is important to recognize that both the R-H's RZF theory and the Hahm's analytic theory on isotope effects are based on the local assumption, where crucial parameters such as q and ε are kept as constants in the radial domain. Consequently, global GTC simulations may not consistently agree with the predictions of the analytic theory. Thus, in order to compare to the analytic theory, we have to use a relatively large device size (R_0) to include sufficient radial wavelengths for the zonal flow while keeping only small variations for global parameters such as q and ε in the simulation domain.

Subsequently, we endeavor to find the minimal number of radial grid points per wavelength required to achieve convergence of the residual zonal flow. In this convergence analysis, we establish the major radius as $R_0 = 166\text{cm}$, yielding the minor radius $a \simeq 278\rho_H$, and set the radial domain size as $\Delta r = 0.36a$. With sufficient wavelengths $N_\lambda = 8$, the resulting normalized radial wavenumber is $k_r\rho_H = 0.5$. Under the assumption of adiabatic electron response, the zonal flow responses at $t = 0$ for various boundary treatments are shown in Fig. 2(a). Notably, significant discrepancies in $\langle\phi\rangle(t=0)$ near the boundaries are observed across different boundary treatments, while the central simulation region maintains consistency. Consequently, we select peaks and troughs around $r = 0.5a$ to measure the residual level and compute its standard deviation by $\sigma = \sqrt{\frac{1}{N} \sum (R_{ZF} - \overline{R_{ZF}})^2}$, where $N = 15$ is the number of the selected radii at peaks and troughs and $\overline{R_{ZF}}$ is the average of these R_{ZF} values.

The findings illustrated in Fig. 2(b) reveal that 50 radial grid points per wavelength suffice to achieve a convergent R_{ZF} at $k_r\rho_H = 0.5$, irrespective of the chosen boundary treatment. At $k_r\rho_H = 1.0$, the waves exhibit greater localization, rendering the boundary treatment more restrictive. As the localized higher k_r waves, whose wavelengths are closer to the scale of the boundaries, are more sensitive to the boundary treatment, and if the simulations of short wavelengths use the same N_λ as long wavelengths, the decreased distance between the boundary and measurement radii makes the measured R_{ZF} more easily affected by the boundary. Therefore, it still manifests a significant effect on R_{ZF} , with a discernible standard deviation. Based on the preceding convergence study, we conclude that the BT-2 boundary treatment is acceptable for 50 radial grid points per wavelength, and the BT-3 boundary treatment is even better as the standard deviation of R_{ZF} is reduced to almost zero.

C. Residual zonal flow without electron polarization

Using the BT-2 boundary treatment and other parameters discussed above, simulations of zonal flow damping for three hydrogen isotopes (H, D, and T) are carried out first without electron polarization, exploring the $k_r\rho_H$ dependence across a range from 0.05 to 7.0.

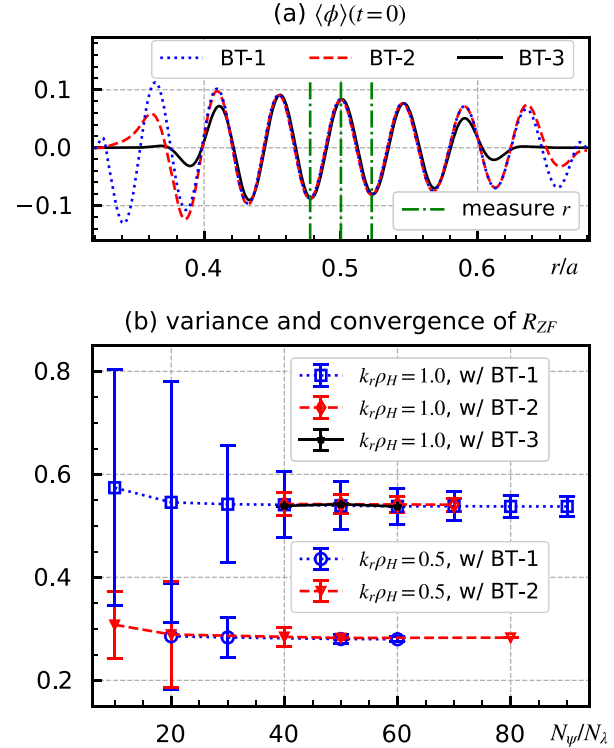


FIG. 2. (a) Initial zonal potential response $\langle\phi\rangle(t=0)$ for three different boundary treatments of the external potential $\langle\phi\rangle_{\text{ext}}$ when $k_r\rho_H = 0.5$, $N_\lambda = 8$. (b) R_{ZF} convergence study for $k_r\rho_H = 0.5$ and $k_r\rho_H = 1.0$ with different boundary treatments of $\langle\phi\rangle_{\text{ext}}$, where N_ψ/N_λ represents the number of radial grid points per wavelength, and the error bar represents the standard deviation of R_{ZF} .

When setting $k_r\rho_H = 0.1$, the temporal evolution of the radial structure of the zonal flow is shown in Fig. 3(a) for the isotope D. From the figure, we observe that the radial wavenumber k_r of the zonal flow in the later time is consistent with the initially prescribed value for $\langle\phi\rangle_{\text{ext}}$. The initial zonal potential $\langle\phi\rangle(t=0)$ oscillates and damps to an asymptotic or residual level at approximately $20R_0/c_{sH}$, which can be effectively considered as $t = \infty$, where c_{sH} is the ion sound speed of isotope H with $c_{sH} = \sqrt{T_e/m_H}$. When setting $k_r\rho_H = 0.5$, the temporal evolution of the zonal flow on the reference flux surface ($r = 0.5a$) is illustrated in Fig. 3(b) for all three hydrogenic isotopes. Compared with $k_r\rho_H = 0.1$, the time required for the wavelength of $k_r\rho_H = 0.5$ to reach the asymptotic level is about at $12R_0/c_{sH}$, which reveals that the damping rate of the oscillatory geodesic acoustic mode (GAM) for $k_r\rho_H = 0.5$ is higher than that for $k_r\rho_H = 0.1$. As shown in Fig. 3(b), the residual level R_{ZF} visibly increases with the isotope mass, implying a significant isotope effect for the wavelength of $k_r\rho_H = 0.5$.

Next, we compare the real frequency and damping rate of GAM for different $k_r\rho_H$ and isotope mass M_i . The real frequency ω_{GAM} remains nearly almost unchanged across different $k_r\rho_H$, from 0.05 to 0.4, as shown in Figs. 4(a) and 4(b). However, ω_{GAM} apparently varies with M_i and heavier isotopes exhibit lower oscillatory frequency. The isotope scaling for the GAM frequency satisfies $\omega_{\text{GAM}} \propto M_i^{-0.5}$. However, the damping rate γ_{GAM} has an exactly opposite conclusion, it is more sensitive to $k_r\rho_H$ rather than M_i . In Figs. 4(a) and 4(c),

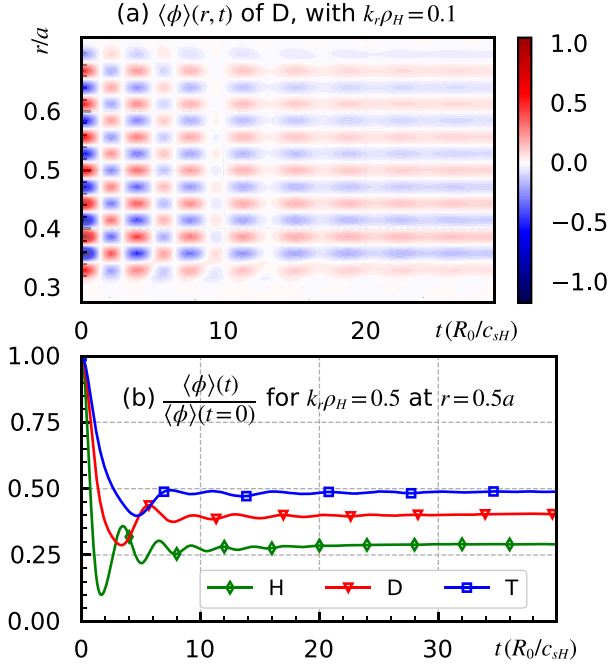


FIG. 3. (a) Time evolution of the radial structure of flux averaged zonal flow for hydrogen isotope D with BT-2, $k_r \rho_H = 0.1$. (b) Time evolution of the zonal flow for three hydrogen isotopes H, D, T on the reference surface $r = 0.5a$ with $k_r \rho_H = 0.5$. c_{sH} is the ion sound speed of isotope H.

despite the overdamped GAM and therefore unreliable measured value of γ_{GAM} for $k_r \rho_H > 0.4$, γ_{GAM} increases linearly with $k_r \rho_H$ from 0.05 to 0.3. The isotope effect of γ_{GAM} is relatively weaker than that of ω_{GAM} , especially for $k_r \rho_H = 0.2$ and 0.3 , which is due to the fact that γ_{GAM} varies more rapidly with $k_r \rho_H$. In the long wavelength region, although only minor difference of γ_{GAM} between three isotopes is observed, the isotope scaling for γ_{GAM} still satisfies $\gamma_{\text{GAM}} \propto M_i^{-0.5}$. The scaling law of ω_{GAM} , $\gamma_{\text{GAM}} \propto M_i^{-0.5}$ is in agreement with the analytic formula presented by Gao *et al.*^{5,6} The coefficients are also compared between our GTC simulation results and the analytic calculations of Gao *et al.*⁵

$$\omega_{\text{GAM}}^2 = \frac{7}{4} \frac{v_{ti}^2}{R_0^2} \left(1 + \frac{46}{49q^2} \right), \quad (30a)$$

$$\gamma_{\text{GAM}} = -0.52 \frac{v_{ti}}{R_0} q^5 \exp \left(-\frac{7q^2}{4} \right), \quad (30b)$$

and Gao *et al.*⁶

$$\omega_{\text{GAM}}^2 = \left(\frac{7}{4} + \tau \right) \frac{v_{ti}^2}{R_0^2} \left[1 + \frac{46 + 32\tau + 8\tau^2}{(7 + 4\tau)^2 q^2} \right], \quad \text{where } \tau = 1, \quad (31a)$$

$$\gamma_{\text{GAM}} = -\frac{\sqrt{\pi}}{2} \frac{v_{ti}}{R_0} \frac{(R_0 \omega_{\text{GAM}})^6}{v_{ti}^6 \left(\frac{7}{4} + \tau \right)} q^5 \exp \left[-\left(\frac{q R_0 \omega_{\text{GAM}}}{v_{ti}} \right)^2 \right], \quad (31b)$$

as illustrated by the dashed and solid black lines, respectively, in Figs. 4(b) and 4(c). For the ω_{GAM} , both the formulas of Gao 2006 and Gao

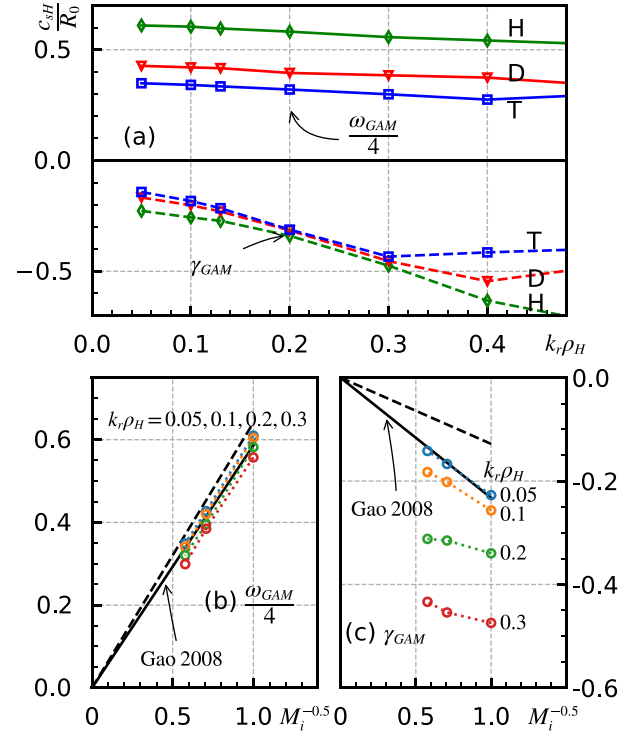


FIG. 4. (a) The real frequency and damping rate of GAM as a function of $k_r \rho_H$ for three different isotopes. The real frequency has a significant isotope effect, while the damping rate has a relatively weak isotope effect. (b) The isotope effect of ω_{GAM} is almost unchanged when increasing $k_r \rho_H$ from 0.05 to 0.3, and it satisfies $\omega_{\text{GAM}} \propto M_i^{-0.5}$. (c) Different $k_r \rho_H$ reveals different isotope effect of damping rate. When $k_r \rho_H = 0.05$, the scaling $\gamma_{\text{GAM}} \propto M_i^{-0.5}$ fully agrees with the analytic results of Gao 2008 [6], as illustrated by the solid black line.

2008 are suitable for the long wavelengths and the intermediate wavelengths, while for the γ_{GAM} , the formula of Gao 2006 underestimates the damping rate and the formula of Gao 2008 is only suitable for the long wavelength limit with the assumption $\gamma_{\text{GAM}} \ll \omega_{\text{GAM}}$. Since the $\mathbf{E} \times \mathbf{B}$ shearing of turbulence is effective only when zonal flow has enough time to affect turbulence eddies, $\omega_{\text{GAM}} < \Delta \omega_T$ (turbulence eddy decorrelation time) should be satisfied for GAM to reduce turbulent transport.¹⁵ It is important to consider the possible role of GAM in regulating transport with this caveat.

After discussing the GAM component of the zonal flow, we turn to focus on comparing the isotope effects on the RZF component. The residual levels R_{ZF} of the three hydrogenic isotopes are plotted as a function of $k_r \rho_H$ and compared with the semi-analytic calculations of X-C in Fig. 5(a). The results show that the residual zonal flow level for H plasmas, i.e., $R_{ZF,H}$, aligns closely with the calculation of X-C for $k_r \rho_H \leq 2.0$, exhibiting a rapid increase in the region $0.1 < k_r \rho_H < 2.0$. In the short-wavelength region with $k_r \rho_H > 2.0$, where the zonal flow becomes more localized, a greater number of N_i and radial grids are required to ensure that the radial simulation domain Δr is set sufficiently large to cover the length scale for ion orbit width, which can be estimated from Fig. 1(a). Moreover, a large number of particles is necessary to ensure the low noise level of R_{ZF} .¹⁶ When the number of particles is increased by changing the

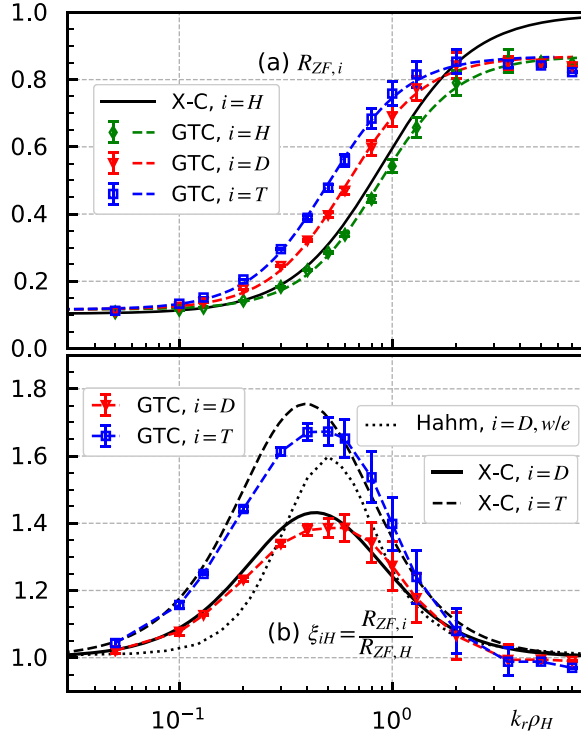


FIG. 5. (a) The residual zonal flow levels of H, D, and T vary with the radial wave-number $k_r \rho_H$ for $q = 1.4, \varepsilon = 0.2$ when only ion polarization is included. The dashed lines represent the non-linear least squares fitting by a modified logistic curve in Eq. (35). (b) R_{ZF} ratios between D, T, and H plasmas as a function of $k_r \rho_H$. GTC simulation results are compared with Hahm's calculation [10] and Xiao-Catto's (X-C) result [12]. The strongest isotope effect is observed at $k_r \rho_H = 0.5$.

poloidal grid number at $r = 0.5a$ from 300 to 2000, the standard deviation of R_{ZF} can be reduced by 46%. Consequently, gyrokinetic simulations for the short-wavelength region require significantly more computational resources. By balancing between computational resources and the numerical accuracy of ion polarization, R_{ZF} from the GTC simulation is a bit lower than the calculation of X-C for high k_r . In the zonal flow simulations for short wavelengths, particularly for $k_r \rho_H > 1.0$, the decay process of GAM becomes overdamped. Although this makes determining the real frequency and damping rate of GAM more challenging, the overdamping of short-wavelength GAM does not significantly impact our primary objective of computing the residual zonal flow level. When varying the ion mass M_i , we observe that the R_{ZF} of D and T shifts toward longer wavelengths due to the increased value of $k_r \rho_i$ for these isotopes at a fixed k_r , which manifests the finite Larmor radius (FLR) effect and finite orbit width (FOW) effect together. This FLR or FOW effect, along with the rapid increase in R_{ZF} for the intermediate wavelengths, plays a crucial role in the observed isotope effect of R_{ZF} in the range of $0.1 < k_r \rho_H < 2.0$.

To further illustrate the isotope effect on the zonal flow, the residual level ratios between D, T, and H plasmas are defined as follows, as suggested by Hahm *et al.*:¹⁰

$$\xi_{iH} \equiv \frac{R_{ZF,i}}{R_{ZF,H}}, \quad (i = D, T). \quad (32)$$

It is suggested that the isotope effect becomes significant when this ratio deviates from unity. The strongest isotope effect is represented by the maximum value of ξ_{iH} , i.e., ξ_{iH}^{max} , and the corresponding $k_r \rho_H$ is denoted as $k_{r,iH}^{max} \rho_H$. The residual level ratio varies with the radial wave-number k_r , as shown in Fig. 5(b). It is evident from this figure that the isotope effect becomes significant within the range of $0.1 < k_r \rho_H < 2.0$, as expected. The strongest isotope effect is observed at $k_r \rho_H = 0.5$. The value of ξ_{DH} is consistent with the semi-analytic calculation of X-C. While showing a similar trend and peak position, the ξ_{DH} obtained from GTC simulations deviates from Hahm's analytical theory¹⁰ for most ion-scale zonal flows. GTC results exhibit an appreciable isotope dependence of RZF for a broader range of $k_r \rho_H$ than the earlier analytic work¹⁰ suggested. The maximum ratio measured, ξ_{DH}^{max} , is 1.4 in the GTC simulation, visibly lower than Hahm's analytic theory of 1.6. It appears that Hahm's theory underestimates ξ_{DH} for long wavelengths within $0.05 < k_r \rho_H < 0.3$, while overestimating ξ_{DH} for intermediate wavelengths within $0.3 < k_r \rho_H < 1.0$. It's important to note that Hahm's analytical formula is based on asymptotic limits of $k_r \rho_H \ll 1.0$ and $k_r \rho_H \gg 1.0$, using Padé approximations for intermediate wavelengths. Therefore, the discrepancies between Hahm's theory and GTC simulation are understandable. Additionally, Hahm's analytical theory includes electron polarization, which is absent in the current GTC simulations. However, the electron polarization is negligibly small for the ion-scale zonal flow and should not cause noticeable effects, as is confirmed by subsequent comparisons between R_{ZF} from GTC simulations and those from the calculations of X-C, in which the electron polarization is fully considered.

Recognizing that the conventional method based on asymptotic matching may not accurately measure the isotope residual zonal flow, we next adopt a more realistic semi-analytic approach. We fit the numerical residual zonal flow level with an analytical function, which may yield more accurate numerical values for the residual zonal flow. The generalized residual zonal flow level as a function of $(q, \varepsilon, k_r \rho_i)$ may be expressed as

$$R_{ZF,i}(q, \varepsilon, k_r \rho_i) = \frac{1}{1 + 1.6 \frac{q^2}{\sqrt{\varepsilon}} (1 - \Theta_i(q, \varepsilon, k_r \rho_i))}, \quad (33)$$

where the factor $\Theta_i(q, \varepsilon, k_r \rho_i)$ is about high order in ε representing the ion FLR and FOW effect, and it satisfies $0 < \Theta_i < 1$ and $\partial \Theta_i / \partial k_r > 0$. The preceding expression may be further transformed to the following expression:

$$R_{ZF,i} = a + \frac{1 - a}{1 + e^{\ln(1 - \Theta_i) - \ln \Theta_i - \ln a}}, \quad (34)$$

where $a = \frac{1}{1 + 1.6 q^2 / \sqrt{\varepsilon}}$ is the Rosenbluth–Hinton residual zonal flow level.⁷ We further note that the preceding expression is similar to the S-shaped logistic function,

$$R_{ZF,i}(x) = a + \frac{b}{1 + e^{c(x+d)}}, \quad (0 < a, b < 1; c < 0). \quad (35)$$

The transitional region of R_{ZF} S-shaped curve is mainly caused by the finite Q or finite FOW effect, where $Q \sim k_r \rho_i q / \varepsilon$. Since the value of $k_r \rho_i$ varies across many orders of magnitude in the assessment, it is more convenient to use an alternative variable x with $x = \ln k_r \rho_i$. Then, take a as the infimum in the long wavelength limit (i.e., R–H residual level), $a + b$ as the supremum in short wavelength limit, and

use parameters c , d to describe the steepness and midpoint of the curve, which mainly affects the difference of R_{ZF} between three different isotopes, then the residual level R_{ZF} in Fig. 5(a) can be fitted by the logistic curve illustrated in Eq. (35). The average relative error of these fitting results is only 3.3%, which confirms the effectiveness of this simple semi-analytic expression.

D. Residual zonal flow with electron polarization

When both trapped and passing electrons are included in the calculation, the zonal flow response is much more sensitive to the boundary treatment of ϕ_{ext} . It is found that the BT-3 best fits our needs for the calculation of the residual zonal flow.

The residual zonal flow level with additional electron polarization as a function of $k_r \rho_H$ is compared between various calculations in Fig. 6(a), including the semi-analytic calculation of X-C,¹² the theory of Wang and Hahm,¹³ GENE simulation of Bustos *et al.*¹¹ and current GTC simulation, for the H plasmas. As shown in the figure, the GTC simulation results are fully consistent with those R_{ZF} from the GENE simulations. When $k_r \rho_H \leq 1.0$, the residual zonal flow R_{ZF} from GTC and GENE simulations with electron response are almost the same as the calculations of X-C and W-H 2009. In the shorter wavelength region with $1.5 \leq k_r \rho_H \leq 11$, R_{ZF} obtained by the GTC and GENE

simulations are much lower than those of X-C and W-H 2009. The underestimation of R_{ZF} in GTC simulations is primarily attributed to the electron drift kinetic model implemented in the GTC code, which ignores the FLR effect for electrons and basically assumes $\rho_e \approx 0$. It is therefore conjectured that the GENE code employs a similar electron model.

For the short wavelength range $1.5 \leq k_r \rho_H \leq 11$, i.e., $0.035 \leq k_r \rho_e \leq 0.26$ or $0.25 \leq k_r \rho_{pe} \leq 1.8$, where $\rho_{pe} = q/\varepsilon \rho_e$ is the electron poloidal gyroradius, by retaining the electron FOW effect and ignoring the electron FLR effect, the R_{ZF} can be written in the following form, according to Eq. (32) of Ref. 12:

$$R_{ZF,i}^{w/e} = \frac{1 - 1/(\sqrt{2\pi} k_r \rho_i)}{1 + 1.6\varepsilon^{3/2} k_r^2 \rho_{pe}^2}, \quad (i = H, D, T). \quad (36)$$

The analytic R_{ZF} in Eq. (36) for isotope H is shown in Fig. 6(a) by the thick solid cyan line, which is very close to the GTC simulation results. Both classical ion polarization and neoclassical electron polarization could contribute to the residual zonal flow level, as shown by Eq. (36). However, for the short wavelength range $1.5 \leq k_r \rho_H \leq 11$, the electron neoclassical polarization dominates the zonal flow response, and thus, R_{ZF} decreases with $k_r \rho_H$ in this range. Let $\partial R_{ZF,i}^{w/e} / \partial k_r = 0$ by using Eq. (36), it is found that the decreasing tendency of R_{ZF} starts at $k_r \rho_H = 4.0$, which is close to the GTC simulation, where the transition occurs around $k_r \rho_H = 3.0$.

Then using three hydrogen isotope species and changing $k_r \rho_H$, once again, we get the difference between $R_{ZF,H}^{w/e}$, $R_{ZF,D}^{w/e}$ and $R_{ZF,T}^{w/e}$, as shown in Fig. 6(a). The isotope effect is still found significant for the intermediate wavelengths. The difference of R_{ZF} between simulations with and without electron polarization is compared for each hydrogen species, which is calculated by $(R_{ZF,i}^{w/e} - R_{ZF,i})/R_{ZF,i}$ and shown in Fig. 6(b). The difference is less than 2% when $k_r \rho_H \leq 2.0$, but it increases significantly when $k_r \rho_H$ is greater than 2.0. It indicates that the electron polarization effect for R_{ZF} in our simulation is much weaker for the long wavelengths and the intermediate wavelengths. Just as the analytic result of Eq. (36) predicts, the electron polarization only becomes important when the radial wavelength decreases to the scale of electron poloidal gyroradius.

E. Residual zonal flow for different q , ε , and M_i

Next, we then conduct a parameter scan of the safety factor q and inverse aspect ratio ε to analyze how $k_{r,iH}^{max} \rho_H$ and ζ_{iH}^{max} vary with the two parameters. Dozens of GTC simulations are performed across the (q, ε) parameter space, covering the ranges $0.8 \leq q \leq 3.4$, $0.1 \leq \varepsilon \leq 0.5$, using the BT-3 boundary treatment. Since the isotope effect on R_{ZF} primarily occurs in the intermediate wavelength region, where electron polarization is negligible, we focus the scan on a $k_r \rho_H$ range from 0.1 to 2.0, retaining only ion polarization in the simulations. First, we outline how we calculate $k_{r,iH}^{max} \rho_H$ and ζ_{iH}^{max} and their dependence on q and ε in this paper. According to W-H 2009 theory, the residual zonal flow becomes

$$R_{ZF,i} = \frac{1}{1 + \chi_{nc,k}^i / k_r^2 \rho_i^2}, \quad (37)$$

where

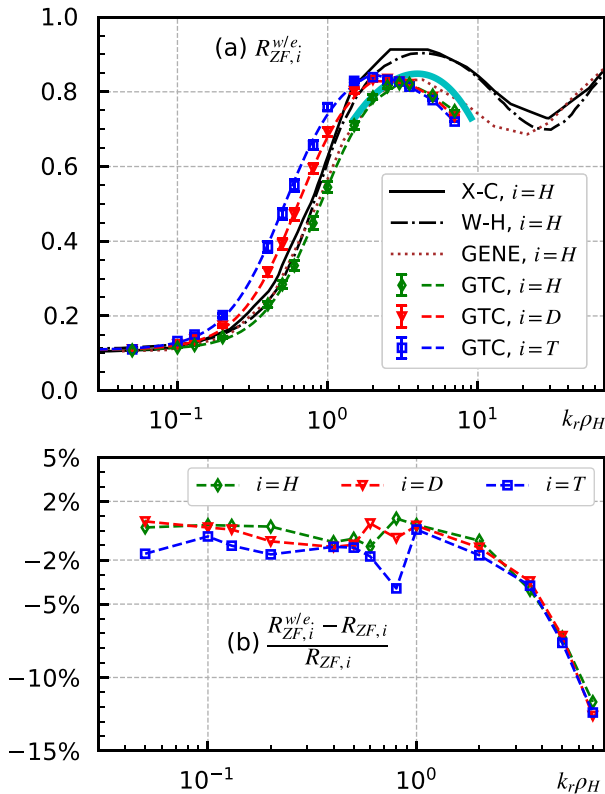


FIG. 6. (a) The R_{ZF} comparison between various analytic theories and numerical calculations for H, D and T, when the electron polarization is included. The solid cyan line represents Eq. (36) for isotope H. (b) Difference of R_{ZF} between the GTC simulations with and without electron polarization.

$$\chi_{nc,k}^i = \left\{ \frac{1}{1.83\epsilon^{3/2}k_r^2\rho_{pi}^2} + \left[1 + \frac{\sqrt{8\epsilon}}{\pi}\Gamma_{tr}' + \left(1 - \frac{\sqrt{8\epsilon}}{\pi} \right)\Gamma_p' \right] \times \frac{1}{1 + k_r^2\rho_i^2} + \sqrt{\frac{\pi^3}{2}}k_r\rho_i \right. \\ \left. \times \left[1 + \frac{\sqrt{8\epsilon}}{\pi}\Gamma_{tr} + \left(1 - \frac{\sqrt{8\epsilon}}{\pi} \right)\Gamma_p \right] \frac{k_r^2\rho_i^2}{1 + k_r^2\rho_i^2} \right\}^{-1}, \quad (38)$$

with $\rho_{pi} = \rho_i q / \epsilon$, $\Gamma_{tr} = 0.915966 / (\sqrt{\pi\epsilon}k_r\rho_{pi})$, $\Gamma_p = 1 / (2\sqrt{2\pi\epsilon}k_r\rho_{pi})$, $\Gamma_{tr}' = 2\Gamma_{tr}/\pi$, $\Gamma_p' = 2\Gamma_p/\pi$. It is noted that there is a difference of the ion neoclassical polarization between Eq. (32) of W-H 2009 and our Eq. (6) by a factor, i.e., $\epsilon_{pol}^{i,nc} = \chi_{nc,k}^i / (k_r^2\rho_i^2) \omega_{pi}^2 / \Omega_{ci}^2$.

The residual level ratio then becomes

$$\xi_{iH} = \frac{k_r^2\rho_H^2 + \chi_{nc,k}^H}{k_r^2\rho_H^2 + \chi_{nc,k}^i / M_i}, \quad (i = D, T), \quad (39)$$

which is the main result of Hahm 2013. Let $\partial \xi_{iH} / \partial k_r = 0$; then, we can solve the equation numerically to obtain

$$k_{r,iH}^{max} = u_k(q, \epsilon, M_i), \quad \text{and} \quad \xi_{iH}^{max} = u_\xi(q, \epsilon, M_i).$$

A quantitative comparison between the preceding theory and GTC simulation is given below. The parameter dependence of $k_{r,iH}^{max} \rho_H$ and

ξ_{iH}^{max} from the preceding theory is plotted in Figs. 7(a) and 7(b) by the contour lines for the deuterium plasma ($i = D$), which shows a same tendency with GTC simulation, and q is shown to be the primary parameter.

As shown in Fig. 7(a), the strongest isotope effect between deuterium and hydrogen plasmas, or largest value of ξ_{DH}^{max} , increases with q and decreases with ϵ , with the effect being more sensitive to q than ϵ . The corresponding wavenumber $k_{r,DH}^{max}$, where the strongest isotope effect on residual zonal flow is observed, is also primarily influenced by q , but shows an opposite dependence on q and ϵ , as seen in Fig. 7(b). It decreases with increasing q , increases with ϵ , and consistently occurs in the intermediate wavelength region.

Then, choosing three different $\epsilon = 0.1, 0.2, 0.3$, a detailed comparison of the variations of ξ_{DH}^{max} and $k_{r,DH}^{max} \rho_H$ as a function of q is carried out in Figs. 7(c) and 7(d). The GTC results of ξ_{DH}^{max} are lower than the prediction of W-H 2009 analytic formula in Eq. (39), which we have seen in Fig. 5(b) for $q = 1.4$, $\epsilon = 0.2$. Within the chosen (q, ϵ) parameter space, it seems that W-H's theory always tends to overestimate the isotope effect of residual zonal flow for the intermediate wavelengths. The advantage of W-H's analytical solution is its ability to distinguish the differences of ξ_{DH}^{max} between different ϵ , while the GTC simulation results can only reveal very slight differences of ξ_{DH}^{max} among the three ϵ . In Fig. 7(d), the comparison of $k_{r,DH}^{max} \rho_H$ shows that the simulation results are consistent with analytic calculation when $q < 2.0$, but have some deviation from analytic theory when $q > 2.0$. In Fig. 8, we show how the ratios $\xi_{TH}^{max} / \xi_{DH}^{max}$ and $k_{r,TH}^{max} / k_{r,DH}^{max}$ vary with

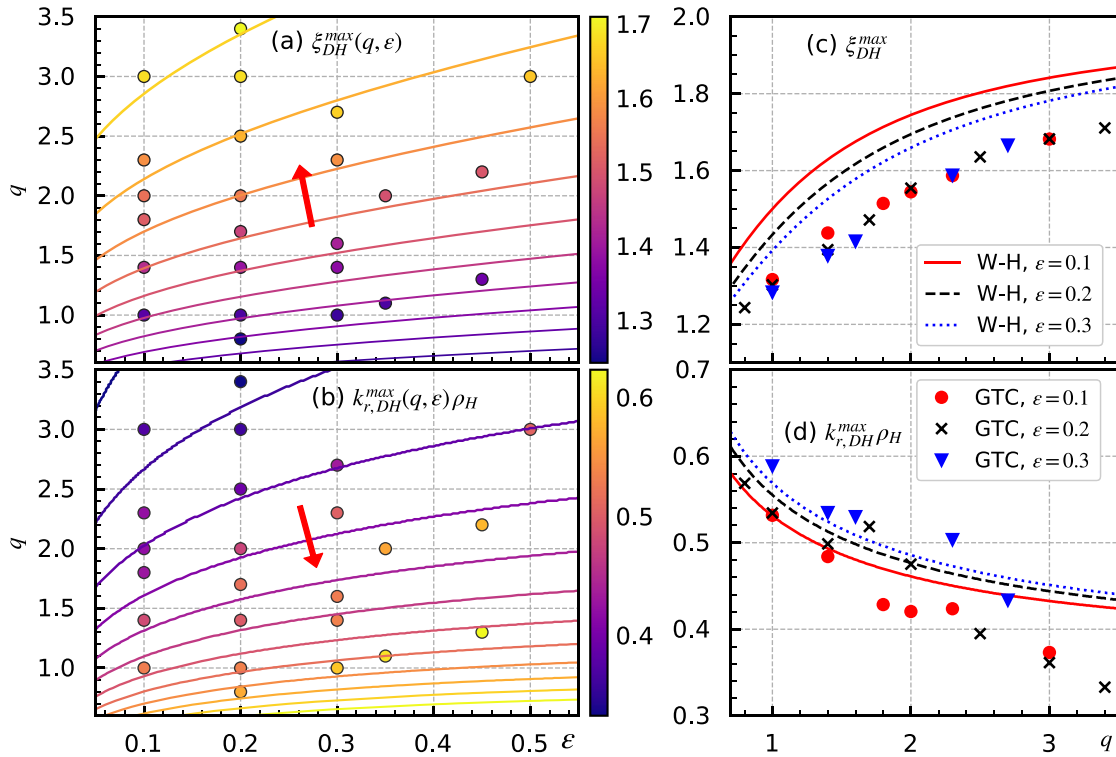


FIG. 7. The strongest isotope effect of residual zonal flow (a) ξ_{DH}^{max} and (b) $k_{r,DH}^{max} \rho_H$ between D and H plasmas for different q and ϵ . The comparison between GTC simulation and the analytic theory of W-H 2009 is presented by the points and contour lines, respectively. The red arrow describes the gradient directions of $\xi_{TH}^{max}(q, \epsilon)$ and $k_{r,DH}^{max}(q, \epsilon)$. The (c) ξ_{DH}^{max} and (d) $k_{r,DH}^{max} \rho_H$ as a function of safety factor q for different $\epsilon = 0.1, 0.2, 0.3$, are compared between GTC and W-H 2009.

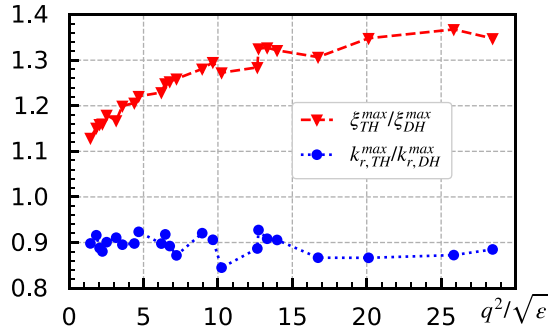


FIG. 8. The comparison of ξ_{iH}^{max} and $k_{r,iH}^{max}\rho_H$ between $i = D$ and $i = T$ for different q and ϵ . The expression $u = q^2/\sqrt{\epsilon}$ is used to transform (ϵ, q) from a two-dimensional parameter space into one dimension. With isotope mass increasing, stronger isotope effect is observed at the longer intermediate wavelength (i.e., lower k_r).

$q^2/\sqrt{\epsilon}$. The ratio $\xi_{iH}^{max}/\xi_{DH}^{max} > 1$ and increases with $q^2/\sqrt{\epsilon}$, so the isotope effect of ξ_{iH}^{max} depends on both (q, ϵ) and M_i , and it is larger for heavier ions. On the other hand, the ratio between $k_{r,TH}^{max}$ and $k_{r,DH}^{max}$ is almost unchanged, i.e., $k_{r,TH}^{max}/k_{r,DH}^{max} \approx 0.9$, for different values of $q^2/\sqrt{\epsilon}$.

The trend of ξ_{iH}^{max} and $k_{r,iH}^{max}\rho_H$ with respect to (q, ϵ) can be further estimated by the transitional behavior of the residual zonal flow in Fig. 5 with its logistic shape fitting Eq. (35). According to the R_{ZF} calculations of Rosenbluth and Hinton⁷ and Xiao and Catto,¹² the residual zonal flow enters the transitional region when the finite Q effect can no longer be ignored, and R_{ZF} starts to increase from $1/(1 + 1.6q^2/\sqrt{\epsilon})$ to 1. Thus, the changes of R_{ZF} in this steep transitional region are

$$\Delta R_{ZF} = 1 - \frac{1}{1 + 1.6q^2/\sqrt{\epsilon}} \approx 1 - \frac{\sqrt{\epsilon}}{1.6q^2}. \quad (40)$$

Meanwhile, note that $Q \sim k_r \rho_H \sqrt{M_i} q / \epsilon$, ($i = H, D, T$), then the larger $\sqrt{M_i} q / \epsilon$ is, the smaller the width of the transitional region $\Delta\{k_r \rho_H\}$ is required to achieve the same amount of ΔQ value, which means that $\Delta\{k_r \rho_H\}$ is inversely proportional to $\sqrt{M_i} q / \epsilon$. Let $Q^0 \sim k_r^0 \rho_H \sqrt{M_i} q / \epsilon$ be a constant representing the Q level where the finite Q effect starts to manifest. Then, the $k_{r,iH}^{max}$ can be estimated to be at the center of the transitional region with the log scaling,

$$k_{r,iH}^{max} = \exp[(\ln(k_r^0 + \Delta\{k_r\}) + \ln(k_r^0)/2] \sim \frac{\epsilon}{q\sqrt{M_i}}. \quad (41)$$

Therefore, $k_{r,TH}^{max}/k_{r,DH}^{max} \sim \sqrt{M_D/M_T} \approx 0.82$. The value of this ratio and its independence of (q, ϵ) is totally consistent with the GTC simulation results.

As the ion FOW effect shifts logistic-shape R_{ZF} curve to longer wavelengths, it is expected that $k_{r,iH}^{max}\rho_H$ decreases for larger ρ_{pi} or q/ϵ . Therefore, the transitional region gets narrower and the logistic curve grows more rapidly due to its exponential feature in this region. Consequently, ξ_{iH}^{max} becomes larger for larger q/ϵ , which is qualitatively proven by the GTC simulation results in Figs. 7(a)–7(d). As shown by this figure, although q affects ξ_{DH}^{max} and $k_{r,DH}^{max}\rho_H$ a lot, but the dependence of q becomes weaker with higher q , especially when $q > 3.0$, which can be found from the sparser contour lines in

Figs. 7(a) and 7(b) or the flatter curves in Figs. 7(c) and 7(d) when q is higher. This picture is consistent with Eqs. (41) and (40) in terms of the q dependence.

Therefore, if assuming that the q value increases monotonically with radius r , the isotope effect of ξ_{iH}^{max} or R_{ZF} in the plasma edge will be stronger than that in the plasma core. Meanwhile, according to Eq. (31b) and Ref. 6, the damping rate of GAM is expected to be weaker for higher q . Then considering zonal flow's regulation effect, we may find more pronounced isotope effect in the high- q region or pedestal top, which was demonstrated by the earlier DIII-D experiments and nonlinear gyrokinetic simulation.¹⁷

IV. CONCLUSION

Isotope effect on turbulent transport provides indispensable information for burning plasma confinement, where the transport may heavily depend on the isotope dependence of zonal flow response. In this paper, we employ gyrokinetic simulations using GTC code to explore the isotope effects on residual zonal flow and GAM for arbitrary wavelengths.

First, we validate the consistency of the gyrokinetic formulation implemented in the GTC code with the gyrokinetic theory on the zonal flow dynamics. We test various boundary treatments for global simulation and find the optimal boundary treatment for the zonal flow problem. Then, we have verified that the real frequency and damping rate of GAM obtained from the GTC simulation are consistent with Gao's analytic theory in the long-wavelength limit. Our simulation results demonstrate qualitative and quantitative agreement with the theory of Hahm 2013 and the semi-analytic calculation of X-C on the residual zonal flow level for a wide range of radial wavelengths. The isotope effect is most pronounced at intermediate wavelengths (i.e., $k_r \rho_i \sim 1$), similar to that found by the analytic theory. However, our findings suggest that Hahm's theory may overestimate the isotope effect of residual zonal flow. This discrepancy may come from the fact that the analytic theory is based on artificial decoupling of FOW and FLR effects for the intermediate wavelengths, and an approximation has to be made to carry out the originally entangled FOW and FLR effects. An S-shaped logistic curve is designed to fit the multi-scale zonal flow response and good agreement is found between the logistic curve and simulation data on the isotope residual zonal flow.

The contribution from electron polarization is negligible in comparison with that from thermal ions at the long and intermediate wavelengths, but it cannot be ignored in the short-wavelength limit (i.e., $k_r \rho_H \geq 1.5$). A detailed discussion has been provided for the difference of R_{ZF} between the GTC simulation and the semi-analytic calculation of X-C in the short-wavelength limit. As the isotope effect occurs only prominently for the intermediate wavelengths, the electron polarization has a negligible effect on the isotope effect of the zonal flow.

It was also found that, at intermediate wavelengths, the isotope effect on zonal flow is influenced by q , ϵ , and M_i , with q and M_i being the primary parameters that significantly affect the strength of the isotope effect. The isotope effect becomes stronger with larger q or heavier isotopes, suggesting that it may be more pronounced in edge plasmas, a phenomenon observed in recent experiments and gyrokinetic simulations.¹⁷

Another implication of the significant isotope effect observed in intermediate-wavelength zonal flow is that electron transport driven by CTM turbulence may exhibit a strong isotope dependence. This is

particularly important for burning plasmas, as α particles are likely to initially utilize electron temperature to excite intermediate-wavelength CTEM turbulence.^{18,19}

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Shengming Li: Conceptualization (equal); Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (equal); Validation (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (equal). **Dingkun Yang:** Data curation (equal); Investigation (supporting); Methodology (supporting); Writing – review & editing (supporting). **Yong Xiao:** Conceptualization (lead); Formal analysis (lead); Funding acquisition (lead); Investigation (equal); Methodology (equal); Project administration (lead); Supervision (lead); Writing – original draft (equal); Writing – review & editing (lead).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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