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Linear gyrokinetic simulation of kinetic infernal mode

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Abstract

Kinetic infernal mode (KIM) is an electromagnetic instability driven by thermal ions in the weak magnetic shear region with a frequency similar to the kinetic ballooning mode (KBM). We present herein the global gyrokinetic simulations of KIM by using the Gyrokinetic Toroidal Code (GTC). It is found that the electromagnetic instability shows a smooth transition from KBM to KIM in both frequency and growth rate when magnetic shear varies from strong to weak, suggesting that KIM and KBM may belong to the same physical mode but exhibit different mode structures due to the influence of magnetic shear. The variation in the spacing of adjacent mode rational surfaces, as revealed by mode structure analysis, is identified as the key mechanism governing the mode transition. Moreover, the magnetic shear and driving source effects are investigated in detail. The simulation results show that KIM prefers to grow on the mode rational surface nearest to the minimum magnetic shear, i.e. where the shear stabilizing effect is weakest, rather than at the location of maximum density gradient or temperature gradient. However, the magnitude of the growth rate is determined by both the magnetic shear and pressure gradient.

Keywords: kinetic infernal mode, gyrokinetic simulation, tokamak

(Some figures may appear in colour only in the online journal)

1. Introduction

Advanced tokamak scenario characterized by the high bootstrap current fraction and high confinement is regarded as one of the possible choices for modern magnetic confinement fusion devices [1–3]. In this scenario, the establishment of internal transport barrier (ITB) could effectively suppress ion transport and improve plasma confinement over a region of plasma core [4, 5]. The ITB region is often associated with

a weak magnetic shear s , (a flattened safety factor (q) profile) [6–10]. Previous studies have verified that the weak magnetic shear could significantly affect the linear and nonlinear dynamics of turbulence [11, 12]. Therefore, it is essential to investigate the effect of weak magnetic shear on fusion plasma instability and associated turbulent transport.

Infernal mode (IM) [13, 14] is a pressure-driven magneto-hydrodynamics (MHD) instability that occurs in the weak or zero magnetic shear region. The normalized plasma pressure β for the infernal instability is found below the critical β value of MHD ballooning modes, where $\beta = 8\pi nT/B^2$ is the ratio between plasma thermal energy and magnetic field energy [14]. Although extensive research has been conducted on the IM in the past two decades [13, 15–19], most of these studies are within the MHD framework. Noting that the kinetic effects, such as finite Larmor radius (FLR) and finite orbit width (FOW) effects, have a stabilizing effect on MHD

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ballooning mode [20, 21], which could potentially act as a similar role for the IM, here we investigate the kinetic IM (KIM) [22].

Previous studies have shown that the traditional ballooning representation is not applicable if the instability is in the weak shear region [23]. More general methods, such as the mode structure decomposition (MSD) [24–26], and Fourier decomposition approaches with weakly coupled poloidal harmonics [27, 28], have been developed for such cases. Nevertheless, to our knowledge, these theoretical tools have not yet been applied to the analysis of KIM.

In this paper, we numerically investigate KIM with global gyrokinetic simulations by GTC code [29, 30]. KIM is obtained by changing the q profile from monotonic to reversed magnetic shear in the high β region while keeping the other parameters the same as those for kinetic ballooning mode (KBM) [31, 32]. The growth rate of KIM is found notably larger than that of KBM, while their real frequencies are comparable [22]. We simulate the KIM under different q profiles and demonstrate here that KIM and KBM may belong to the same mode physically. Moreover, we conduct an extensive investigation on the relationship between KIM and the q profile, as well as the free energy source, namely the temperature gradient (R/L_{Ti}) and density gradient (R/L_{ni}). The simulation findings show that weak shear and mode rational surface positions are crucial factors in the formation of KIM eigenmode, whereas the driving free energy sources determine the magnitude of its growth rate.

This paper is organized as follows. In section 2, the gyrokinetic simulation model in the Gyrokinetic Toroidal Code (GTC) is described. Section 3 presents the mode transition from ion temperature gradient (ITG) to KBM and KIM in normal and reversed shear q profiles, respectively, as β increases. The growth rates of KIM and IM with toroidal mode number n are also discussed. In section 4, the relationship between KBM and KIM is explored through a scan of the magnetic shear s , and a qualitative formula is derived to distinguish between these two instabilities. Furthermore, section 5 discusses the relationship between KIM and q profile, as well as the free energy driving sources (R/L_{ni} or R/L_{Ti}). Finally, in section 6, we provide a summary of the KIM simulation results and discuss future work.

2. GTC formulation for KIM simulation

Our simulations are based on the electromagnetic gyrokinetic model for collisionless plasmas implemented in the GTC code. The gyrokinetic equation describes the plasma dynamics in 5D phase space, where gyrocenter position $\mathbf{X}$, magnetic moment μ , and parallel velocity $v_{\parallel}$ serve as its coordinates [30, 33–35]:

$$\frac{df_{\alpha}(\mathbf{X}, \mu, v_{\parallel}, t)}{dt} = \left(\frac{\partial}{\partial t} + \dot{\mathbf{X}} \cdot \nabla + \dot{v}_{\parallel} \frac{\partial}{\partial v_{\parallel}} \right) f_{\alpha} = 0 \quad (1)$$

$$\dot{\mathbf{X}} = v_{\parallel} \frac{\mathbf{B}}{B_0} + \langle \mathbf{v}_E \rangle_c + \mathbf{v}_c + \mathbf{v}_g \quad (2)$$

$$\dot{v}_{\parallel} = -\frac{1}{m_{\alpha}} \frac{\mathbf{B}^*}{B_0} \cdot (\mu \nabla B_0 + Z_{\alpha} \nabla \langle \phi \rangle_c) - \frac{Z_{\alpha}}{m_{\alpha} c} \frac{\partial \langle A_{\parallel} \rangle_c}{\partial t}. \quad (3)$$

In the preceding equations, the particle species label $\alpha = i, e$ represents thermal ions and electrons, respectively. In addition, f_{α} is the distribution function, m_{α} is the particle mass and Z_{α} denotes the particle charge. $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}$ where $\mathbf{B}_0 = B_0 \mathbf{b}$ is the equilibrium magnetic field, and $\delta \mathbf{B} = \nabla \times (\langle A_{\parallel} \rangle_c \mathbf{b}_0)$, $\langle \cdots \rangle_c = 1/2\pi \int \cdots \delta(\mathbf{X} + \boldsymbol{\rho} - \mathbf{x}) d\vartheta_c d\mathbf{x}$ is the gyro-phase average from particle coordinate to gyrocenter coordinate. $\mathbf{B}^* = \mathbf{B}_0^* + \delta \mathbf{B} = \mathbf{B}_0 + \frac{B_0 v_{\parallel}}{\Omega_{\alpha}} \nabla \times \mathbf{b}_0 + \delta \mathbf{B}$, with the gyrofrequency Ω_{α} . $A_{\parallel}$ and ϕ are the perturbation of electrostatic and vector potentials. μ represents the magnetic moment. $\langle \mathbf{v}_E \rangle_c$, $\mathbf{v}_c$, $\mathbf{v}_g$ is the $E \times B$ drift velocity, curvature drift velocity and grad-B drift velocity, respectively, which are given by

$$\langle \mathbf{v}_E \rangle_c = \frac{c \mathbf{b}_0 \times \nabla \langle \phi \rangle_c}{B_0} \quad (4)$$

$$\mathbf{v}_c = \frac{v_{\parallel}^2}{\Omega_{\alpha}} \nabla \times \mathbf{b}_0 \quad (5)$$

$$\mathbf{v}_g = \frac{\mu}{m_{\alpha} \Omega_{\alpha}} \mathbf{b}_0 \times \nabla B_0. \quad (6)$$

We use particle-in-cell (PIC) approach and delta-f method to solve this gyrokinetic-Maxwell plasma system [36]. The ion weight is defined as $w_i = \delta f_i / f_i$, where $f_i = f_{0i} + \delta f_i$ with f_{0i} representing the equilibrium part and δf_i the perturbed part. The perturbed part δf_i satisfies,

$$\begin{aligned} \frac{d\delta f_i}{dt} = & - \left(v_{\parallel} \frac{\delta \mathbf{B}}{B_0} + \mathbf{v}_E \right) \cdot \nabla f_{0i} + \left(\mu \frac{\delta \mathbf{B}}{B_0} \cdot \nabla B_0 \right. \\ & \left. + Z_{\alpha} \frac{\mathbf{B}^*}{B_0} \cdot \nabla \langle \phi \rangle_c + \frac{Z_i}{c} \frac{\partial \langle A_{\parallel} \rangle_c}{\partial t} \right) \frac{1}{m_i} \frac{\partial f_{0\alpha}}{\partial v_{\parallel}}. \end{aligned} \quad (7)$$

Then we can get

$$\begin{aligned} \frac{dw_i}{dt} = & (1 - w_i) \left[- \left(v_{\parallel} \frac{\delta \mathbf{B}}{B_0} + \mathbf{v}_E \right) \cdot \frac{\nabla f_{0i}}{f_{0i}} + \left(\mu \frac{\delta \mathbf{B}}{B_0} \cdot \nabla B_0 \right. \right. \\ & \left. \left. + Z_{\alpha} \frac{\mathbf{B}^*}{B_0} \cdot \nabla \langle \phi \rangle_c + \frac{Z_i}{c} \frac{\partial \langle A_{\parallel} \rangle_c}{\partial t} \right) \frac{1}{m_{\alpha} f_{0\alpha}} \frac{\partial f_{0\alpha}}{\partial v_{\parallel}} \right]. \end{aligned} \quad (8)$$

The fluid-kinetic hybrid model is adopted in GTC [30, 37]. In this model, the electron response is divided into a lowest order adiabatic part as well as a higher order non-adiabatic part, or a kinetic part. The continuity equation of adiabatic electron density is given by [30]

$$\begin{aligned} \frac{\partial \delta n_e}{\partial t} + B_0 \mathbf{b}_0 \cdot \nabla \left(\frac{n_{e0} \delta u_{e\parallel}}{B_0} \right) + B_0 \mathbf{v}_E \cdot \nabla \left(\frac{n_{e0}}{B_0} \right) \\ - n_{e0} (\mathbf{v}_{*e} + \mathbf{v}_E) \cdot \frac{\nabla B_0}{B_0} = 0 \end{aligned} \quad (9)$$

where $\mathbf{v}_{*e} = \mathbf{b}_0 \times \nabla (\delta P_{e\parallel} + \delta P_{e\perp}) / (n_{e0} m_e \Omega_e)$, $\delta P_{\parallel} = \int d\mathbf{v} m v_{\parallel}^2 \delta f_e$, $\delta P_{\perp} = \int d\mathbf{v} \mu B_0 \delta f_e$, $n_{e0} = \int d\mathbf{v} f_{e0}$. The electron parallel fluid velocity results from the parallel Ampere's law

$$n_{e0} e \delta u_{e\parallel} = \frac{c}{4\pi} \nabla_{\perp}^2 \delta A_{\parallel} + n_{i0} Z_i \delta u_{i\parallel}. \quad (10)$$

The vector potential is obtained from

$$\frac{\partial \delta A_{\parallel}}{\partial t} = \nabla_{\parallel} (\delta \phi_{\text{eff}} - \phi) \quad (11)$$

ϕ is obtained from the gyrokinetic Poisson's equation [38],

$$\frac{Z_i n_i}{T_i} (\phi - \tilde{\phi}_i) = \sum_{\alpha=i,e} Z_{\alpha} \delta n_{\alpha} \quad (12)$$

where $\tilde{\phi}$ and δn_{α} are the double gyro-phase average potential and the gyrocenter density, respectively. As shown in equation (13), the effective scalar potential $\delta \phi_{\text{eff}}$ can be derived by integrating the leading order of $\omega/k_{\parallel} v_{\parallel}$ in electron gyrokinetic equation due to the small electron-to-ion mass ratio [30, 37], where ψ and α denote the poloidal flux label and the magnetic field line label, respectively.

$$\frac{e \delta \phi_{\text{eff}}}{T_e} = \frac{\delta n_e^0}{n_{e0}} - \frac{\partial \ln n_{e0}}{\partial \psi} \delta \psi - \frac{\partial \ln n_{e0}}{\partial \alpha} \delta \alpha. \quad (13)$$

The gyrokinetic Vlasov equation together with the parallel Ampere's law, gyrokinetic Poisson's equation, form a closed system for gyrokinetic electromagnetic simulation.

To consider kinetic electronic effect, the distribution function can be divided into $f_e = f_{0e} + \delta f_e = f_{0e} + \delta f_e^{(0)} + \delta h_e$, the weight is defined as $w_e = \delta h_e / f_e$. The gyrokinetic equation for perturbed part of electron distribution function can be written as

$$\begin{aligned} \frac{d \delta h_e}{dt} = f_{0e} \left[- \left(\frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \frac{\delta f_e^{(0)}}{f_{0e}} - \mathbf{v}_E \cdot \nabla \ln f_{0e} \Big|_{v_{\perp}} \right. \\ \left. + \frac{e}{T_e} \mathbf{v}_d \cdot \nabla \langle \phi \rangle_c \right]. \end{aligned} \quad (14)$$

Rewrite equation (14) in terms of particle weight,

$$\begin{aligned} \frac{dw_e}{dt} = \left(1 - \frac{\delta f_e^{(0)}}{f_{0e}} - w_e \right) \left[- \mathbf{v}_E \cdot \nabla \ln f_{0e} \Big|_{v_{\perp}} - \frac{\partial}{\partial t} \frac{\delta f_e^{(0)}}{f_{0e}} \right. \\ \left. - \mathbf{v}_d \cdot \nabla \frac{\delta f_e^{(0)}}{f_{0e}} + \frac{e}{T_e} \mathbf{v}_d \cdot \nabla \langle \phi \rangle_c \right] \end{aligned} \quad (15)$$

integrating δh_e in velocity space, the first order correction of electron kinetic effect can be resolved,

$$\delta n_e^0 = \delta n_e - \int \delta h_e d\mathbf{v}. \quad (16)$$

To manifest the important role of zonal flow, we usually divide the potential ϕ and parallel vector potential into to a zonal component and non-zonal component, i.e. $\phi = \delta \phi + \langle \phi \rangle$, $A_{\parallel} = \delta A_{\parallel} + \langle A_{\parallel} \rangle$ [30, 39]. However, for present linear instability investigations, we can ignore the contribution from the zonal component and restrict $\phi = \delta \phi$, $A_{\parallel} = \delta A_{\parallel}$. A more detailed derivation can be found in [30].

3. KIM instability

In this section, the electromagnetic instabilities under different conditions and models are displayed. We first present linear gyrokinetic simulations for the mode transition in normal and reserved magnetic shear cases with a concentric circular equilibrium specifically as a benchmark. Two distinct profiles are considered: a monotonous normal-shear q profile given by $q(r) = 0.85 + 2.18(r/a)^2$, and a reversed-shear q profile given by $q(r) = 1.76 + 10(r/a - 0.5)^2$, where a represents the minor radius.

The remaining parameters are configured according to the Cyclone Base Case (CBC) parameters: the major radius $R_0 = 83.5$ cm, the minor radius $a = 0.36R_0$; the temperature profile is modeled as $T_s = \text{Exp}[-T_1 \tanh((r - r_0)/T_2)]$, with $T_1 = 0.3a/L_r$, $T_2 = 0.3a$, $r_0 = 0.5a$ representing the reference magnetic surface; similarly, the density profile is expressed as $n = \text{Exp}[-n_1 \tanh((r - r_0)/n_2)]$ with $n_1 = 0.3a/L_n$, $n_2 = 0.3a$. Such plasma profile settings ensure that $R/L_{ti}(r_0) = 6.92$ and $R/L_n(r_0) = 2.22$, where $L_{ti} = -(\text{dln} T_i / \text{dr})^{-1}$ denotes the temperature gradient and $L_n = -(\text{dln} n / \text{dr})^{-1}$ denotes the temperature gradient, respectively [22, 40].

KIM and KBM can be simulated with identical parameters except for q profile. Figure 1 displays the linear growth rate and real frequency of these instabilities with β for a toroidal mode number $n = 10$ under monotonic and reversed q profiles. It is evident that as the normalized plasma pressure β increases, the real frequency of the instability represented by the blue curve in figure 1(a) undergoes a jump, indicating the mode has changed from ITG to KIM. Both KIM and KBM exhibit consistent trends in their real frequencies and growth rates with increasing β . While the growth rate of KIM is significantly higher than that of KBM, and the threshold β for KIM is much lower than that for KBM.

Figure 2 illustrates the electrostatic potential $\delta \phi$ of these instabilities. The 2D mode structures in figure 2(a) indicate that both KIM and KBM are located in the bad curvature region. However, as shown in figure 2(b), KBM displays a typical ballooning mode structure, whereas KIM is primarily localized near the $q_{\min}$ region with weak magnetic shear. With the features of KIM and KBM, the relationship between these electromagnetic modes will be discussed in section 4.

Existing analytical theories for IMs are primarily applicable to low toroidal mode numbers (n) [41, 42], whereas kinetic effects—critical for intermediate to high n regimes—significantly influence KIM dynamics. This discrepancy complicates direct comparison between our kinetic simulations of KIM and conventional IM theory. To address this gap, we simulate the IM using the ideal MHD framework in the GTC [43], enabling a controlled benchmark against analytical predictions.

As shown in figure 3, the dispersion relations of the IM (MHD) and KIM exhibit strong agreement in the long-wavelength limit, with nearly identical growth rates. This convergence suggests that kinetic effects diminish at larger scales, aligning with the theoretical expectation that MHD approximations hold validity for long-wavelength perturbations. While

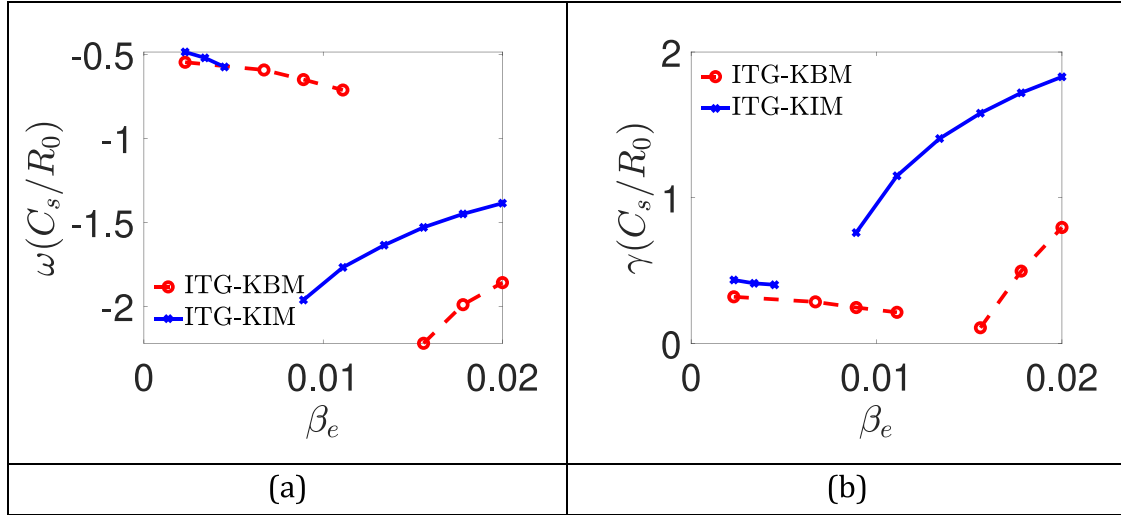


Figure 1. The mode transition from ITG to KBM/KIM as β increases for $n = 10$: (a) real frequency, (b) linear growth rate.

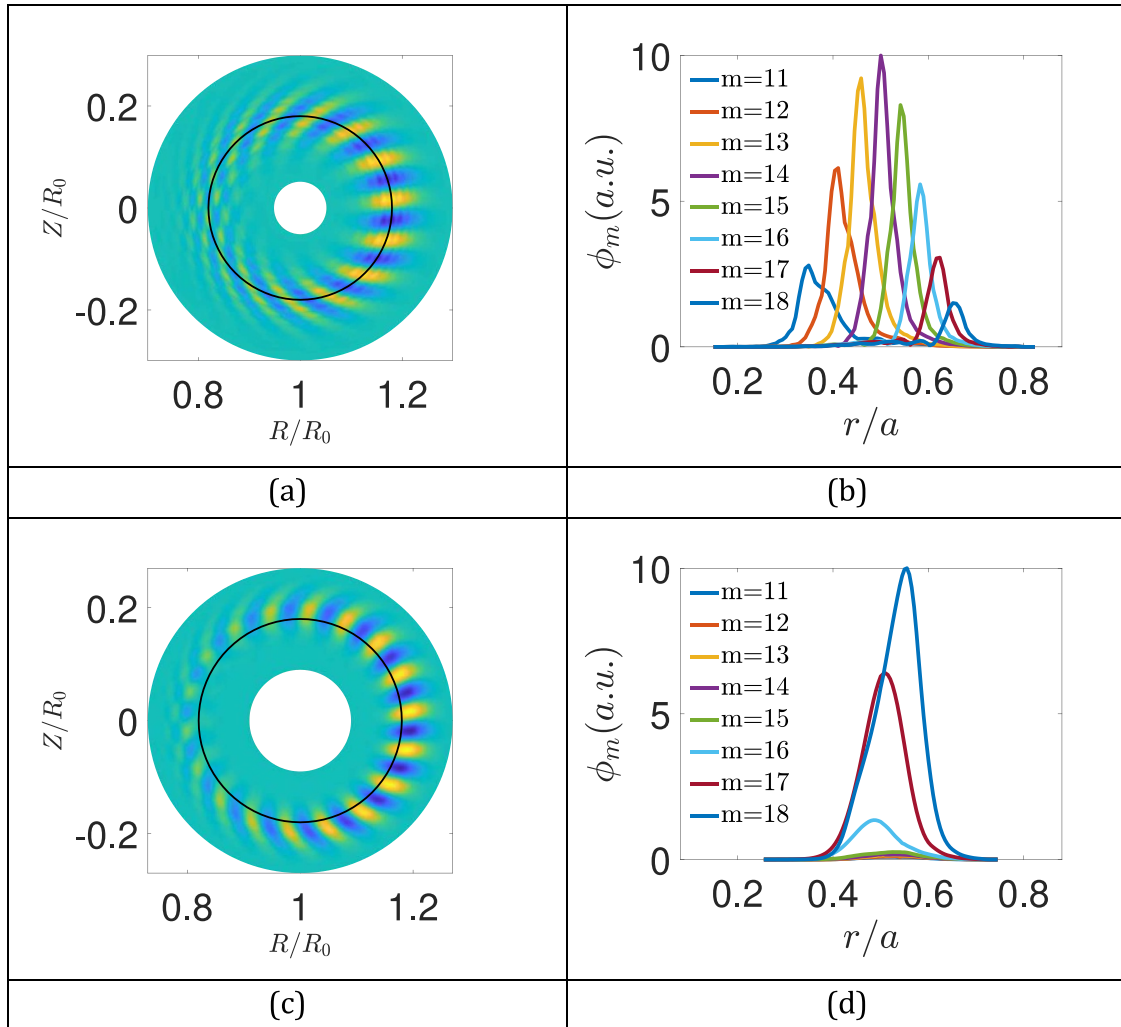


Figure 2. Mode structures of KBM and KIM. (a) 2D mode structure of KBM, (b) poloidal harmonics of KBM, (c) 2D mode structure of KIM, (d) poloidal harmonics of KIM. The black circle line in (a) and (c) represents the position of $r/a = 0.5$.

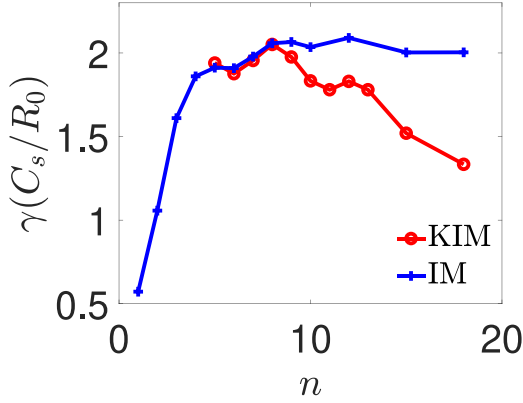


Figure 3. Linear growth rate γ of KIM and IM with toroidal mode number n at $\beta = 2\%$.

in the high n region, there is significant difference between the growth rate of KIM and IM, suggest that kinetic effect could suppress the infernal instability when $n > 12$ ($k_\theta \rho_i \sim 0.33$) [44]. This result underscores the crucial role of kinetic effects on the KIM instability. Note that it is challenging to simulate low n ($n < 5$) electromagnetic instability (KIM) for a gyrokinetic code since $k_\theta \rho_i \sim 0.07$ when $n = 3$, whereas the gyrokinetic code is optimized for cases with $k_\theta \rho_i \sim O(1)$ and $k_\parallel \ll k_\perp$.

4. Relationship between KIM and KBM

In last section, we demonstrate that KIM and KBM differ in mode structure, growth rate, and the threshold of β for ITG-KIM (KBM) transition. However, KIM and KBM exhibit nearly identical real frequencies, and the relationship of their real frequencies and growth rate with β are similar. In this section, we would explore the connection between KIM and KBM. More specifically, KIM and KBM are both electromagnetic instabilities driven by pressure gradient, but with different magnetic shears defined as $s = rq'/q$. When s is moderate or strong ($s \sim O(1)$), the pressure gradient driven electromagnetic instability is KBM. However, when s is weak, i.e. $|s| \ll 1$, the pressure gradient driven electromagnetic instability has been identified as KIM. Meanwhile, the plasma pressure has to be set sufficiently large to excite electromagnetic instability, thus we set $\beta = 0.02$, $R/L_{ti} = 6.92$, and $R/L_n = 2.22$. To investigate the relationship between KIM and KBM, we systematically vary the magnetic shear s at the diagnosis surface, and measure the frequency and growth rate of the electromagnetic instabilities.

Figure 4 illustrates the equilibrium profiles used in the simulation. In figure 4(a), the q profile is set as $q(r) = 1.76 + c_1(r/a - 0.5)^2$, where the slope of q profile or magnetic shear changes as we vary c_1 from -1.8 to 2.0 . For comparison purposes, it is important to note that the safety factor q at $r = 0.5a$ remains constant at 1.76 . Figure 4(b) illustrates the corresponding magnetic shear $s(r = 0.5a)$ ranging from -0.5 to 0.5 for different q profiles in figure 4(a). Figures 4(c) and (d)

present the temperature and density profiles, as well as their gradient, respectively.

The dependence of instability frequencies and growth rates on magnetic shear (s) across various q profiles is systematically analyzed in figure 5. As shown in figure 5(a), the growth rate exhibits a non-monotonic dependence on s , peaking near $s = 0.2$ before declining. Concurrently, the real frequency exhibits a non-monotonic trend similar to the growth rate in figure 5(b).

To identify whether the instability is KIM or KBM under varying magnetic shears, figure 6 presents four representative radial mode structures of the perturbative potential $\delta\phi$ for different magnetic shears. In these subfigures, the red dashed line illustrates the radial profile of the safety factor q , which is displayed in orange on the right side of figure 6 for reference. The solid lines, rendered in different colors, depict the radial mode structure for different poloidal mode numbers (m) while the vertical dashed lines of the same colors denote the corresponding rational surfaces with scales indicated on the left axis of each figure.

In figure 6(a), the magnetic shear is negative while in figure 6(b), the q profile is a constant straight line at 1.76 , independent of r/a , indicating zero magnetic shear across the simulation domain. The magnetic shears are positive in figures 6(c) and (d) with figure 6(c) showing weak magnetic shear and figure 6(d) displaying strong magnetic shear. As shown in figures 6(a) and (c), the unstable modes do not grow precisely at the corresponding rational surfaces. Additionally, the radial mode structures in figure 6(b) indicates that the instability localizes at the point with strongest driven mechanism, as no rational surfaces exist. These characteristics confirm that the instabilities in figures 6(a)–(c) are KIM. Conversely, in figure 6(d), where the magnetic shear is sufficiently large ($s \sim 0.4$), the radial profile of different poloidal harmonics for one particular toroidal mode number (n) exhibits typical characteristics of a ballooning mode structure, suggesting the presence of KBM.

Our parameter scan of magnetic shear reveals a smooth spectral transition from KIM to KBM. Unlike the ITG-KBM transition marked by discontinuous jumps in real frequency, the KIM-KBM transition demonstrates continuous variations in both real frequency and linear growth rate (see figure 5) under our equilibrium. Hence, we hypothesize that KIM and KBM belong to the same mode. It is worth noting that GTC is an initial value code, and more definitive conclusions require verification by an eigenvalue gyrokinetic code. Critically, if KIM and KBM were distinct modes, their eigenfrequencies would need to diverge significantly—a requirement incompatible with the overlapping spectral trends in figure 5. Thus, despite methodological limitations, our hypothesis retains robust empirical support.

We conduct a simple qualitative analysis of the instabilities with their mode rational surface distribution. We can treat $\delta = \Delta_{rs}/l_e$ as the weak shear condition analogous to the strong plasma gradient condition described in [23]. Here l_e represents the full width at half maximum (FWHM) of the radial mode structure envelope, while $\Delta_{rs} \sim 1/(nq')$ denotes the distance between adjacent rational surfaces of the most

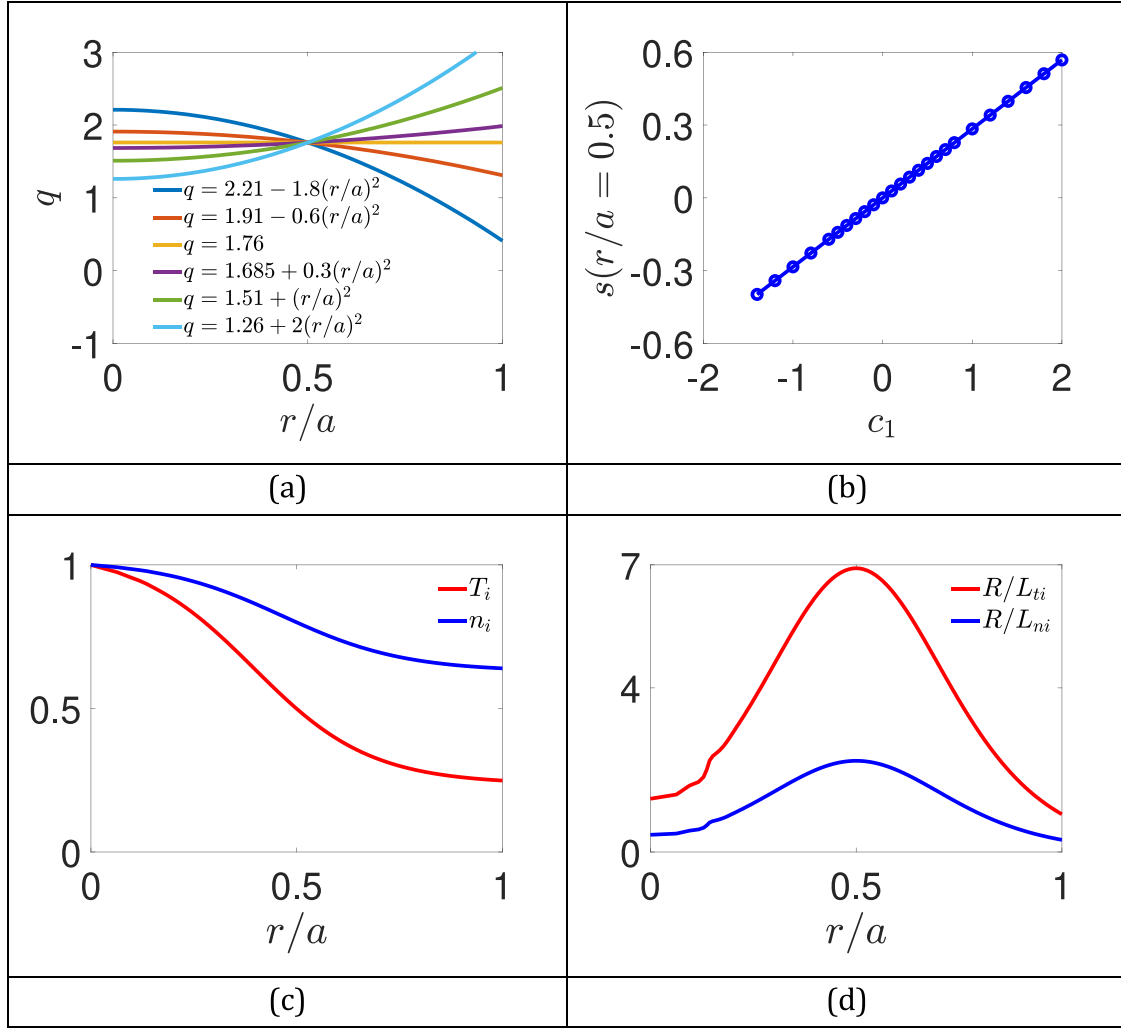


Figure 4. (a) Typical safety factor (q) profiles for different values of c_1 , (b) magnetic shear (s) at the diagnostic surface ($r = 0.5a$), (c) density and temperature profiles, (d) the density and temperature gradients.

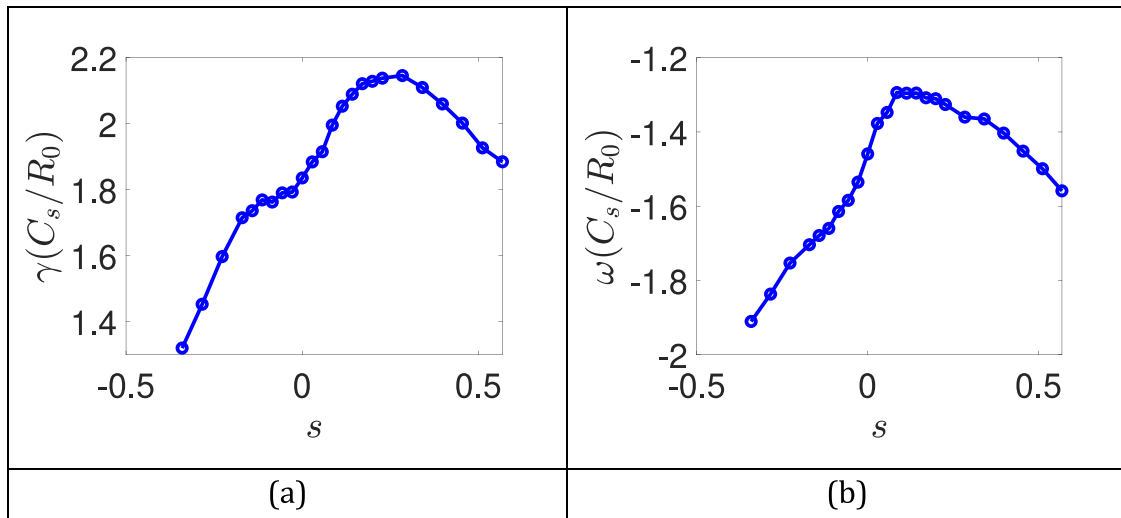


Figure 5. Linear electromagnetic instabilities for different values of magnetic shear (s) depicted in figure 2 with $\beta = 0.02$: (a) growth rate, (b) real frequency.

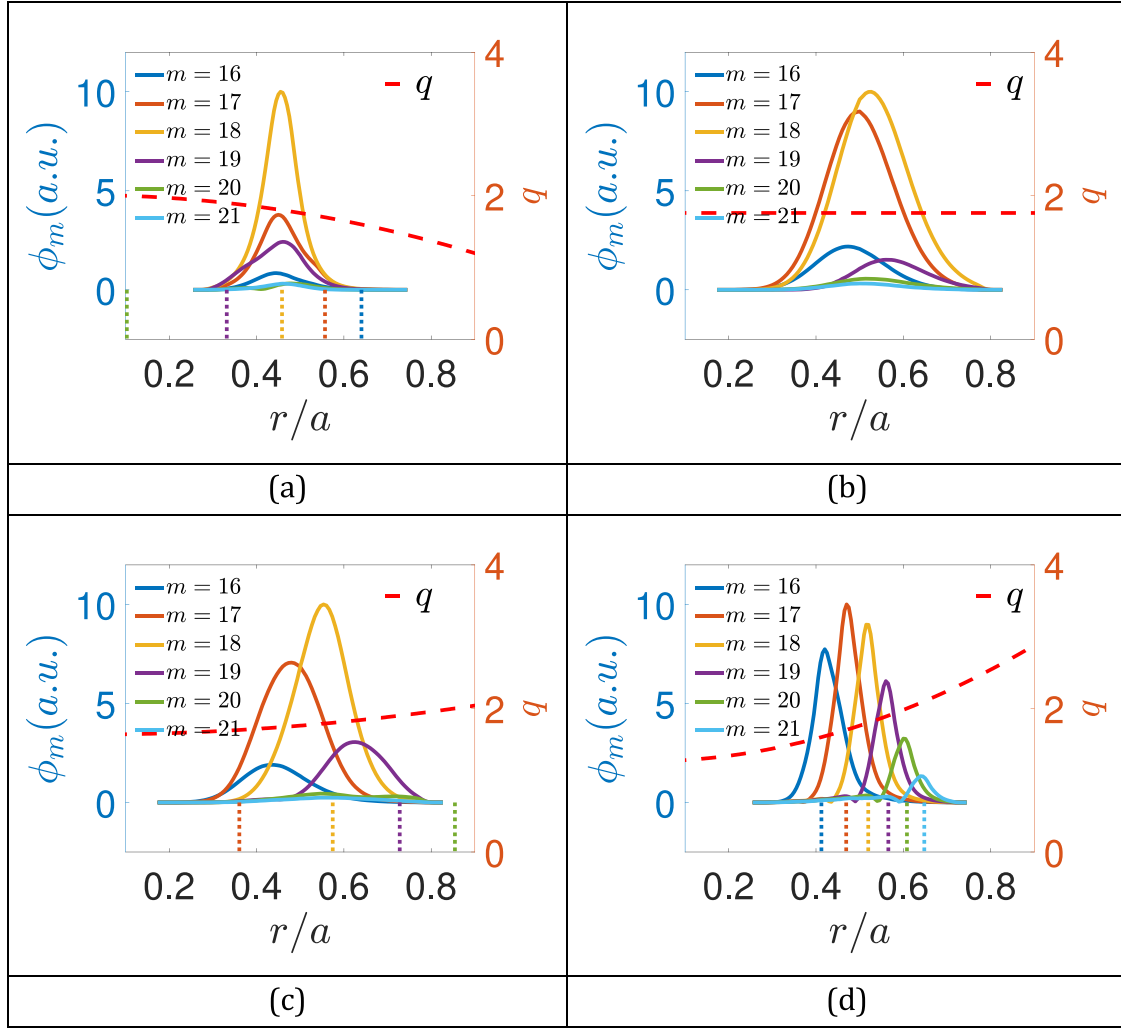


Figure 6. Radial mode structure of electromagnetic instability for different magnetic shears: (a) $s = -0.142$, (b) $s = 0$, (c) $s = 0.142$, (d) $s = 0.5682$, with $\beta = 0.02$.

unstable poloidal harmonics. When $\delta \ll 1$, the instability classified as KBM, when $\delta \sim 1$ or $\delta > 1$, it corresponds to KIM.

Figure 7 illustrates the l_e and Δ_{rs} of the instabilities depicted in figures 6(c) and (d). In figure 7(b), for the instability in figure 6(d), the FWHM is approximately $0.19r/a$, and the distance between adjacent rational surfaces is $0.05r/a$, yielding $\delta \sim 0.26 \ll 1$. This indicates that the distance between rational surfaces is negligibly small compared to the radial envelope width. The use of translational invariance in the ballooning representation is appropriate for treating the coupled poloidal harmonics. In this large s regime, the instability is clearly KBM and exhibits a characteristic ballooning mode structure.

As s decreases, the global effect becomes essential for the weak magnetic shears, modes with different poloidal mode numbers gradually move away from the position of their corresponding rational surfaces, leading to the breakdown of the ballooning mode structure. When s becomes sufficiently small, as shown in figure 7(a), the FWHM of the instability in figure 6(c) is $l_e \sim 0.21r/a$, and $\Delta_{rs} \sim 0.21r/a$, resulting in $\delta \sim 1$. In this case, the variation of the radial envelope

of the instability becomes comparable to the variation of a specific poloidal harmonic radial profile with $(nq - m)$. Therefore, translational invariance is broken and the unstable mode becomes KIM. In summary, the weakness of magnetic shear leads to the sparsity of mode rational surface, which thereby enables the excitation of KIM.

In addition, according to the weak shear condition, we can further discuss the relationship between KIM-KBM transition and toroidal mode number n . For a given q profile, $\Delta_{rs} \sim 1/(nq')$ decreases with increasing n , leading to a smaller δ . Consequently, the electromagnetic instability is more likely to exhibit a ballooning mode structure. To verify this analysis, simulations were performed for different toroidal mode numbers n with the same safety factor profile. Figure 8 presents the radial mode structures for instabilities with $n = 6$ and $n = 10$ under $q(r) = 1.36 + 1.6(r/a)^2$. Comparing figures 8(a) and (b), the instability at $n = 10$ exhibits a more distinctly ballooning-mode structure. Therefore, increasing the toroidal mode number n reduces the threshold in s for the KIM-KBM transition.

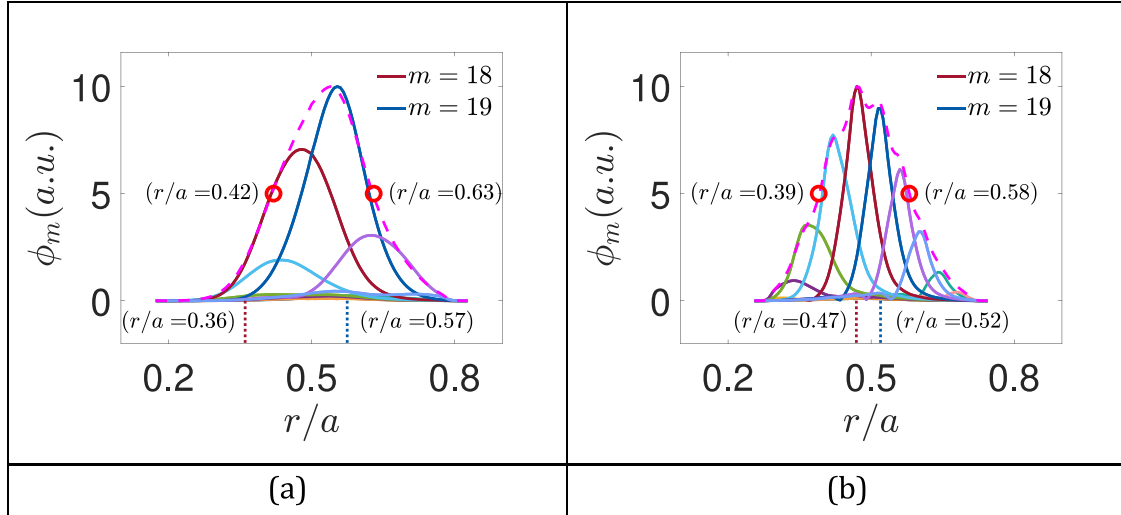


Figure 7. The radial envelope and full width at half maximum of the instabilities shown in figure 6. Panel (a) corresponds to the radial mode structure from figure 6(c), while panel (b) corresponds to the radial mode structure from figure 6(d). In both panels, the dashed lines represent the envelope of the radial mode structure.

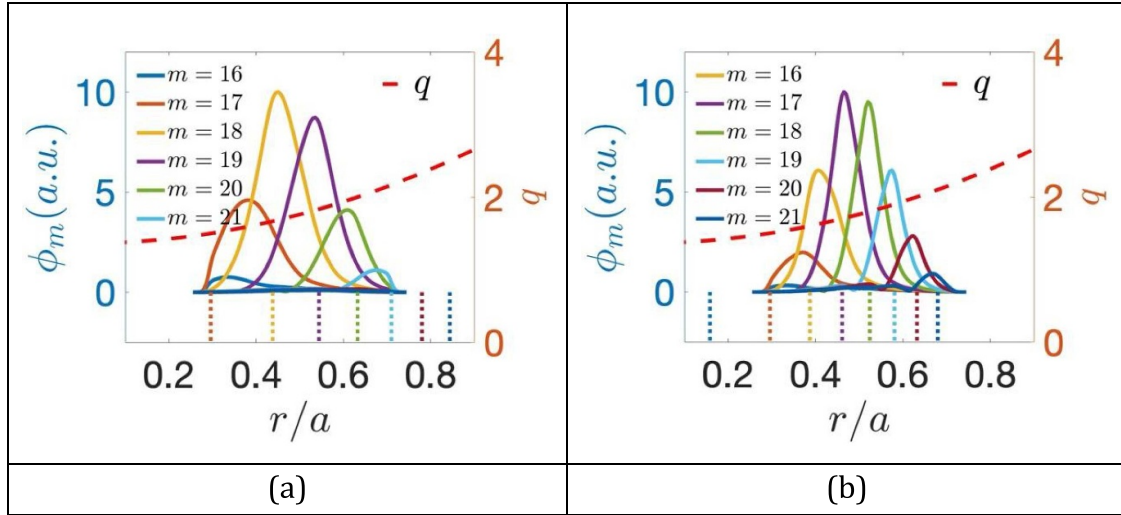


Figure 8. Radial mode structure of the electromagnetic instability for different toroidal mode number n with the same q profile: (a) $n = 6$, (b) $n = 10$.

5. Physics analysis for KIM

Simulation results presented in the preceding section illustrate that KIM is strongly influenced by the q profile. In general, q profile can influence field line bending term in the vorticity equation, which represents a stabilizing effect according to the energy principle [24], while the instability driving source $d\beta/dr$, which is related to the temperature and density gradients of ions and electrons, i.e. $R/L_{ii,e}$, $R/L_{ni,e}$ respectively, is accounted for by the ballooning interchange term in the vorticity equation [20, 21, 24, 45]. In this section, we will examine how these factors, including the shape of q profile, and driving source $d\beta/dr$, can affect the linear properties of KIM separately, as well as determine whether the driving source profile or the weak magnetic shear plays the dominant role in the growth of KIM.

In figures 9(a) and (b), the reversed q profiles are given as: $q(r) = 1.76 + 2(r/a - 0.5)^2$ and $q(r) = 1.76 + 20(r/a - 0.5)^2$ respectively. Within the q profile shown in figure 9(a), the mode structure shown in figures 9(c) and (e) illustrate that the mode does not appear at the q_{min} position indicated by the black line but locates at the mode rational surface nearest to zero shear point. This is because at the mode rational surface, $k_{||} \sim nq(r_m) - m = 0$, the stabilizing effect of landau damping is minimal, which causes the KIM more unstable. In figure 9(b), the q profile is steep, thus the strongest poloidal harmonics of KIM shown in figures 9(d) and (f) are closer to the q_{min} position compared to it shown in the figure 9(c). This result implies that the KIM is correlated with mode rational surface.

Next, we investigate the effect of magnetic shear on KIM in more detail. Figure 10 presents some cases demonstrating

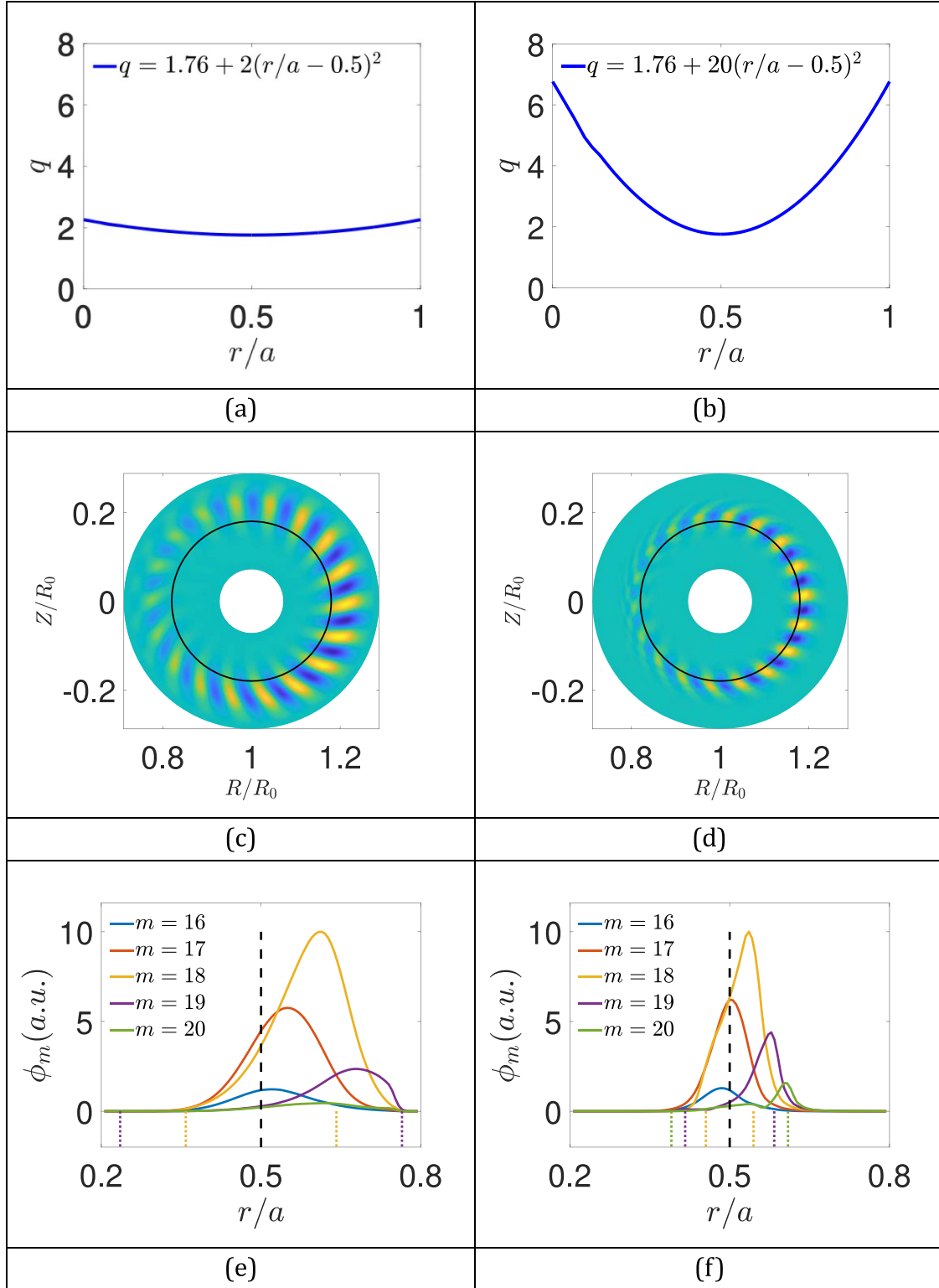


Figure 9. 2D poloidal mode structure and poloidal harmonics for KIM instability with two different reversed-shear q profiles. In (a), (c) and (e), $q(r) = 1.76 + 2(r/a - 0.5)^2$. In (b), (d) and (f), $q(r) = 1.76 + 20(r/a - 0.5)^2$. The middle radial point is given by the black line in (c)–(f) representing the location of $r/a = 0.5$.

the constraining effect of magnetic shear s on KIM. In these simulations, $R/L_{ni,e}$, $R/L_{ti,e}$ remain unchanged as shown in figure 4(d) while q profiles vary. All q profiles are set as

reversed shear with the same minimum value (q_{min}), but the radial positions of q_{min} are shifted from left to right across the simulation domain, as indicated by the red dashed lines.

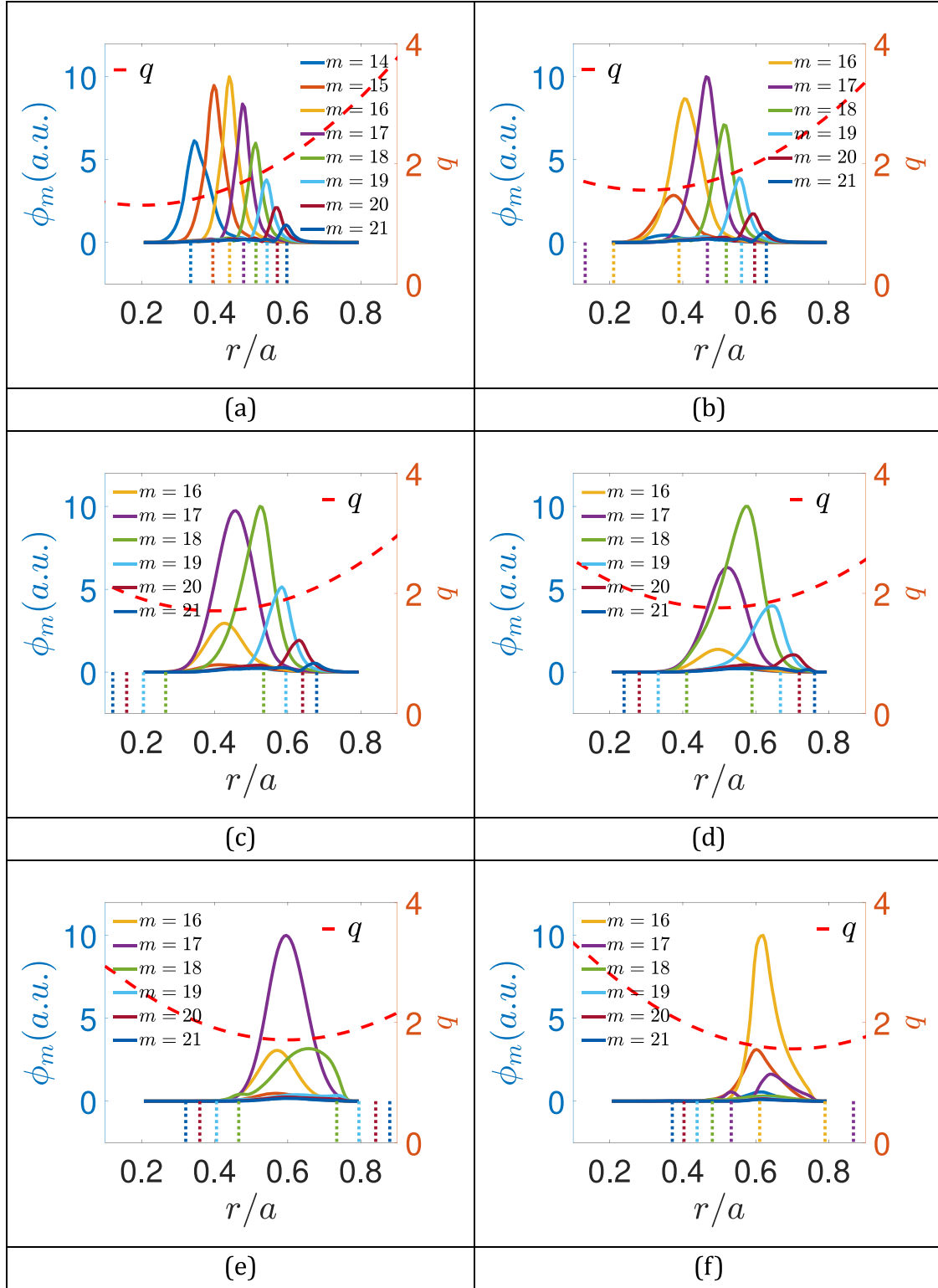


Figure 10. Radial model structure of KIM/KBM for different radial locations of $q_{\min}$, where $\beta = 0.02$ (a) $r_{q_{\min}} = 0.2a$, (b) $r_{q_{\min}} = 0.3a$, (c) $r_{q_{\min}} = 0.4a$, (d) $r_{q_{\min}} = 0.5a$, (e) $r_{q_{\min}} = 0.6a$, (f) $r_{q_{\min}} = 0.7a$.

In figures 10(a) and (b), the q profile is positioned near the left boundary, making it appear nearly monotonically increasing within the simulation domain. In figure 10(a), the average distance between adjacent rational surfaces is $\Delta_{rs} \sim 0.04R_0$,

and the FWHM of radial mode structure is $l_e \sim 0.19R_0$, resulting in $\delta \sim 0.21 \ll 1$. Similarly, $\delta \sim 0.27 \ll 1$ in figure 10(b). These small δ values indicate that the electromagnetic instabilities in these two cases are KBM.

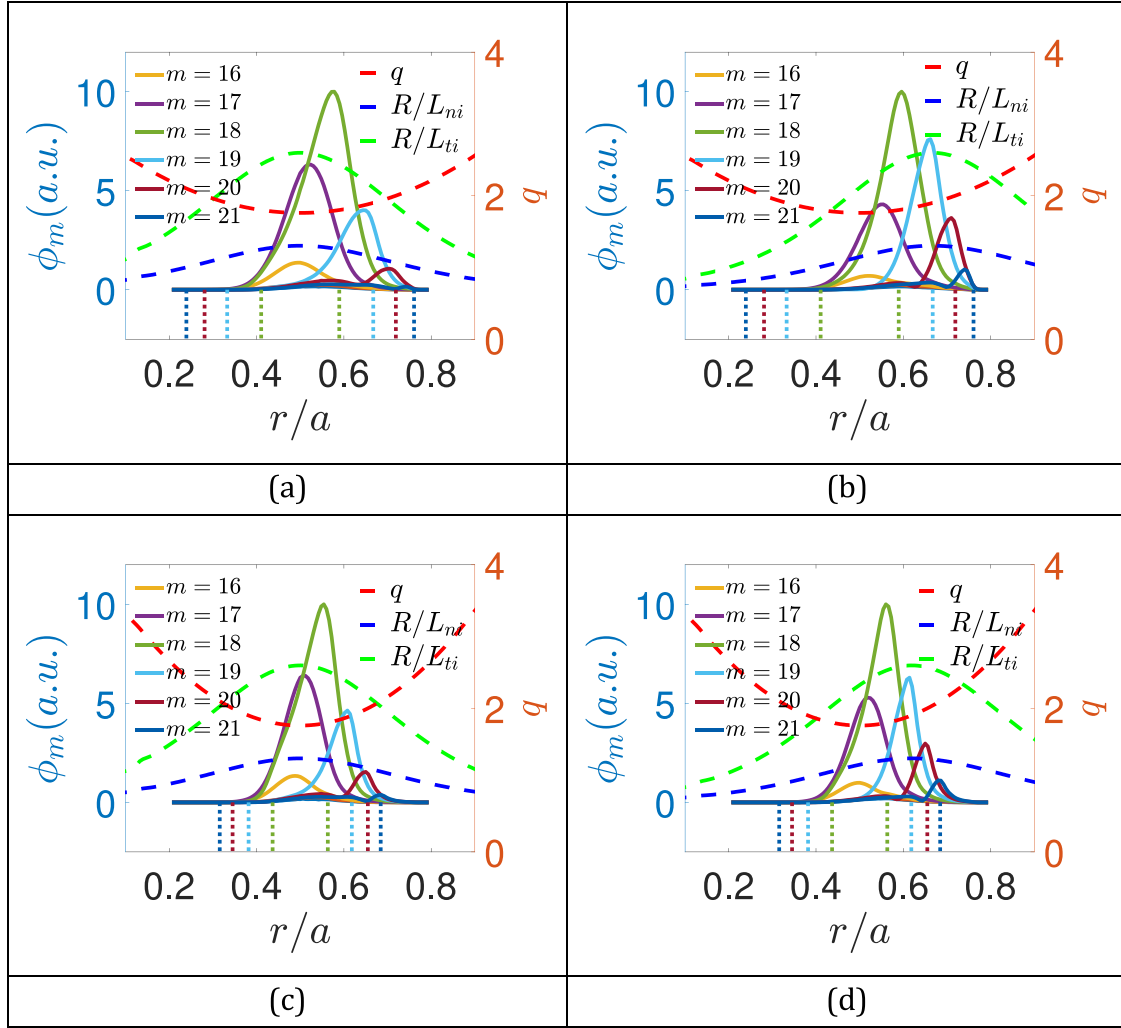


Figure 11. Radial mode structure for various peak locations of R/L_{ti} and R/L_{ni} with different q profiles. In (a) and (b), $q(r) = 1.76 + 2(r/a - 0.5)^2$. In (c) and (d), $q(r) = 1.76 + 20(r/a - 0.5)^2$. The peak locations of R/L_{ti} and R/L_{ni} is (a) $r_{\text{peak}} = 0.5a$, (b) $r_{\text{peak}} = 0.6673a$, (c) $r_{\text{peak}} = 0.5a$, (d) $r_{\text{peak}} = 0.6183a$.

In contrast, in figures 10(c)–(f) $\delta \sim 1$ confirms that the instabilities are KIM. In these cases, the poloidal harmonics radial profiles of the instabilities move along with the weak shear region, which is due to the weakest stabilizing effect at the location of q_{min} . These results demonstrate that the weak magnetic shear is crucial for the formation of KIM.

Combining the results of figures 9 and 10, it is evident that weak magnetic shear and the distribution of mode rational surface have a significant influence on the formation of KIM. KIM is excited in the weak shear region, and normally, the most unstable mode occurs at the mode rational surface nearest to the position of q_{min} . This characteristic may facilitate further theoretical investigation of the linear instability of KIM.

Finally, we investigate the effects of the driving source, especially the temperature and density gradients, on KIM, and determine whether the driving source profile or the weak magnetic shear plays the dominant role in constraining KIM. Figure 11 presents four cases with two different q profiles. In figures 11(a) and (b), the q profile is $q(r) = 1.76 +$

$2(r/a - 0.5)^2$, while in figures 11(c) and (d), the q profile is $q(r) = 1.76 + 5(r/a - 0.5)^2$. In these figures, the green and blue dashed lines represent the temperature gradient $R/L_{Ti,e}$, and the density gradient $R/L_{ni,e}$, respectively, and correspond to the scale on the left side of the plot for reference. To ascertain whether the driving source profile or the weak magnetic shear dominates in the formation of KIM, consideration must be given to the selection of the peak location of the driving source. This is crucial to guarantee that the rational surface nearest to the peak diverges from the rational surface closest to the zero magnetic shear point.

Thus, in figures 11(a) and (c), the peaks of $R/L_{Ti,e}$ and $R/L_{ni,e}$ locate at $r/a = 0.5$, while in figures 11(b) and (d), the peaks are positioned at the rational surface second closest to the $s = 0$ position. It is obvious that from figures 11(a) and (b), as well as figures 11(c) and (d), the radial mode structure position of KIM remains unchanged even when the position of driven changed. These simulation results suggest that, similar to the non-resonant modes discussed by Ishida Y. *et al* [22],

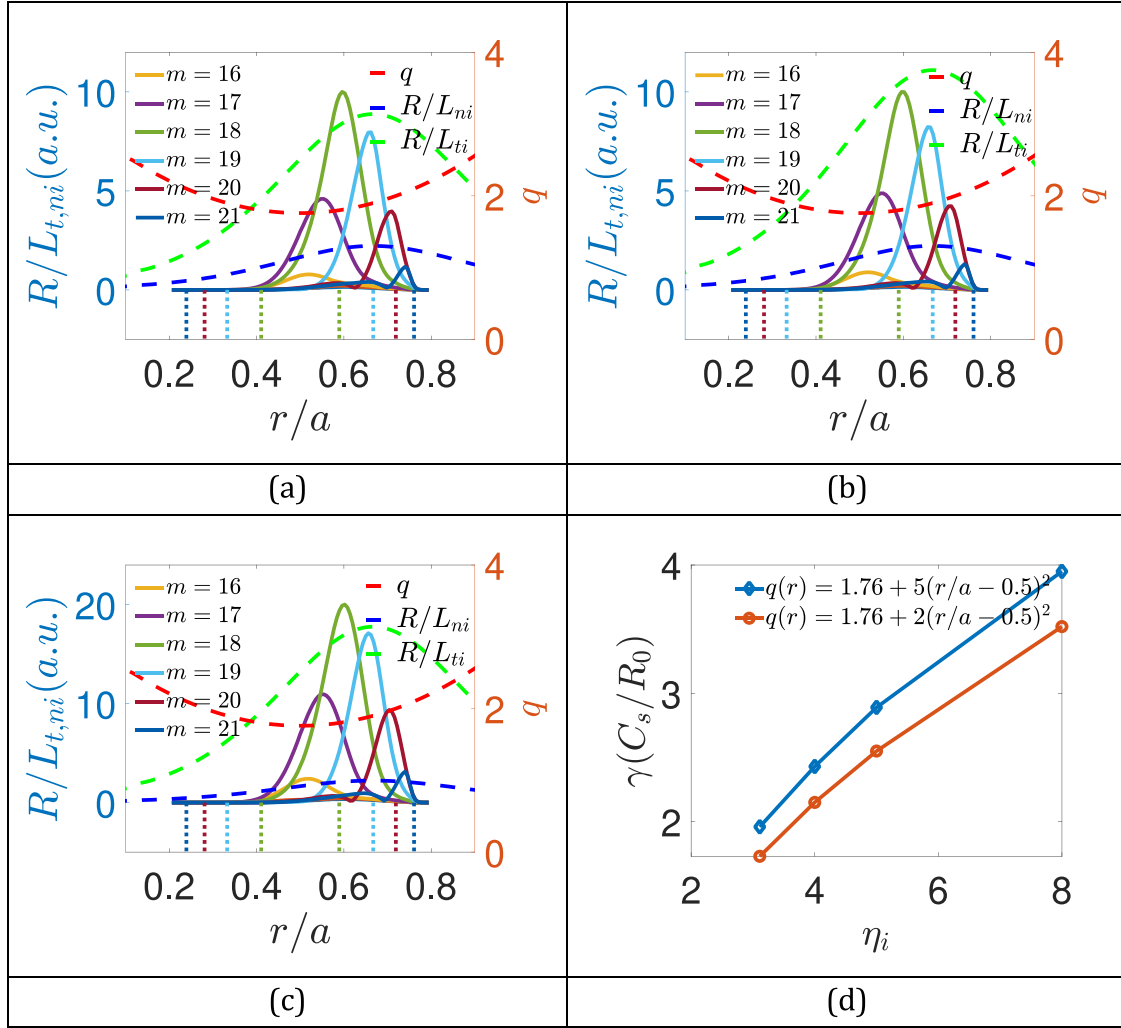


Figure 12. Radial mode structure for different η_i : (a) $\eta_i = 4$, (b) $\eta_i = 5$, (c) $\eta_i = 8$, (d) the KIM growth rate varies with η_i for various q profiles.

the resonant modes also do not depend on the location of the driving source peak but are instead influenced by weak magnetic shear.

To further explore the influence of driving source on KIM, stronger pressure gradients such as $R/L_{ti,e} = 8.88, 11.1, 17.76$, are selected as driving sources. Figure 12(d) depicts that, for each q profile, the growth rate of KIM increases with the parameter $\eta_i \equiv L_{ti}/L_{ni}$, especially when $\eta_i = 8$, the KIM growth rate nearly doubles comparing to that for $\eta_i = 3$. Figures 12(a)–(c) shows the radial mode structures of KIM under different driving gradients. The simulation results demonstrate that even with a broader variation in the driving pressure gradient, the radial position of the most unstable KIM mode aligns closely with mode rational surface closest to q_{min} , in agreement with the results shown in figure 11.

Combining the conclusions of figures 11 and 12, it can be inferred that the stabilizing effect from field-line bending, specifically magnetic shear, exerts a stronger influence on the location of KIM compared to the driving source. And the driving free energy source solely impacts the growth rate of KIM

without inducing any changes in the radial location of its mode structure.

6. Summary

This study investigates linear properties of KIM using global gyrokinetic simulations with the GTC code. By changing the magnetic shear, we observe a continuous transition in both the linear growth rate and real frequency from KIM to KBM. This phenomenon implies that KIM and KBM could be the same mode physically. A qualitative method has been developed to distinguish KBM and KIM based on the ratio of the distance between adjacent mode rational surfaces and radial mode envelope δ [23]. For KBM, $\delta \ll 1$, the distance between adjacent mode rational surfaces is small compared to the radial envelope, and the global effect is weak due to the strong magnetic shear, resulting in a typical ballooning mode structure. In contrast, when $\delta \geq 1$, KIM locates at the weak shear region, the variation of radial envelope is non-negligible

as the distribution of rational surface is sparse, leading to the breakdown of translational invariance.

Additionally, the influence of the mode rational surface, the magnetic shear, and the driving source R/L_{Ti} , R/L_{ni} on KIM has been investigated comprehensively. It is found that KIM can only be excited in the weak shear region, and the most unstable mode occurs at the mode rational surface nearest to the position of q_{min} . And the driven term, pressure gradient primarily affects the growth rate of KIM without altering its radial peak position.

The IM has been observed in JT-60U and ASDEX-U [46, 47]. In ITER hybrid scenarios, where the plasma β is relatively high and the core magnetic shear is weak, KIM are likely to influence turbulence transport in this region. Thus, the future work will focus on the nonlinear simulation of KIM under experimental equilibrium.

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