

# Nonlinear Particle Simulation of Ion Cyclotron Waves in Toroidal Geometry

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**Abstract.** Global particle simulation model has been developed in this work to provide a first-principles tool for studying the nonlinear interactions of radio frequency (RF) waves with plasmas in tokamak. In this model, ions are considered as fully kinetic particles using the Vlasov equation and electrons are treated as guiding centers using the drift kinetic equation with realistic electron-to-ion mass ratio. Boris push scheme for the ion motion has been developed in the toroidal geometry using magnetic coordinates and successfully verified for the ion cyclotron and ion Bernstein waves in global gyrokinetic toroidal code (GTC). The nonlinear simulation capability is applied to study the parametric decay instability of a pump wave into an ion Bernstein wave side band and a low frequency ion cyclotron quasi mode.

## INTRODUCTION

Understanding the nonlinear physics (e.g., parametric decay instabilities, nonlinear Landau damping etc.) associated with radio frequency heating and current drive is crucial for plasma confinement and steady state operation of fusion experiments. It is well known that plasma current can be driven effectively by externally launched radio frequency waves (e.g., lower hybrid wave, ion cyclotron wave and ion Bernstein waves).

The eigenvalue solvers like TORIC [1] and AORSA [2] have been widely used to study high frequency waves such as the ion cyclotron wave, ion Bernstein wave, and lower hybrid wave. However, this method does not capture the crucial nonlinear physics. To address these crucial nonlinear physics we have developed a toroidal PIC simulation model based on the initial value approach [3-4]. Nonlinear phenomena of the RF waves have been studied in the slab geometries with particle codes such as GeFi [5] and Vorpall [6].

## PHYSICS MODEL

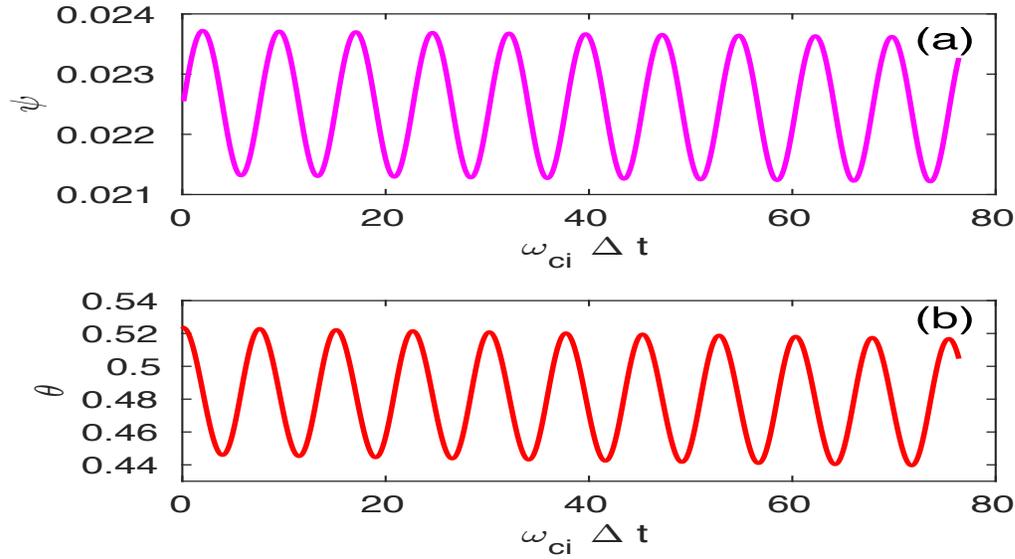
Ion dynamics is described by the six dimensional Vlasov equation,

$$\left[ \frac{\partial}{\partial t} + \vec{v} \cdot \nabla + \frac{Z_i}{m_i} (\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial}{\partial \vec{v}} \right] f_i = 0 \quad (1)$$

where  $f_i$  is the ion distribution function,  $Z_i$  is the ion charge, and  $m_i$  is the ion mass. The evolution of the ion distribution function  $f_i$  can be described by the Newtonian equation of motion in the presence of self-consistent electromagnetic field as follows

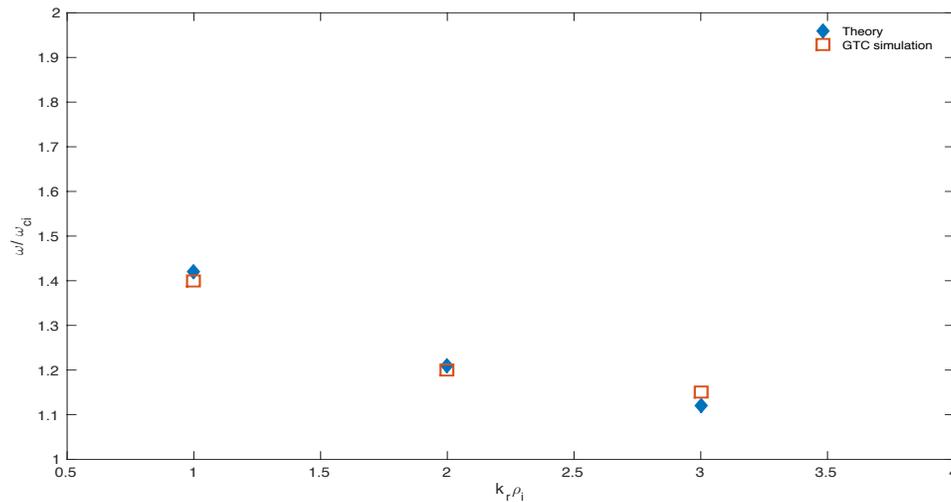
$$\frac{d}{dt} \vec{r} = \vec{v} \quad \text{and} \quad \frac{d}{dt} \vec{v} = \frac{Z_i}{m_i} [\vec{E} + \vec{v} \times \vec{B}] \quad (2)$$

In our simulation we use toroidal magnetic coordinate system  $(\psi, \theta, \zeta)$ . The magnetic field of this system is  $\vec{B} = I\nabla\theta + g\nabla\zeta = q\nabla\psi \times \nabla\theta - \nabla\psi \times \nabla\zeta$ . In our simulation we compute the marker particle trajectory [Eq. (2)] by the time centered Boris push method [3,7]. To verify this integrator we carried out the single particle motion in this toroidal geometry. Figure 1 shows the verification of the cyclotron motion of ion in the toroidal geometry.



**FIGURE 1.** Verification of cyclotron motion in toroidal geometry. Time history of (a) poloidal flux function and (b) geometric poloidal angle.

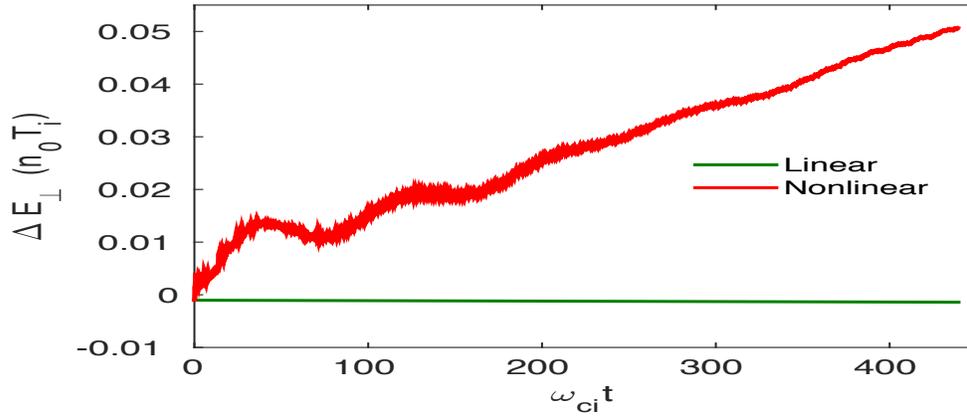
Secondly we have verified the dispersion relation of the IBW in the toroidal geometry. We use an artificial antenna to excite these modes and to verify the frequency in our simulation. Figure 2 demonstrates a good agreement between the analytical and GTC simulation results of the IBW frequency.



**FIGURE 2.** Comparison of ion Bernstein wave dispersion relation between analytic results and GTC simulations for the first harmonic.

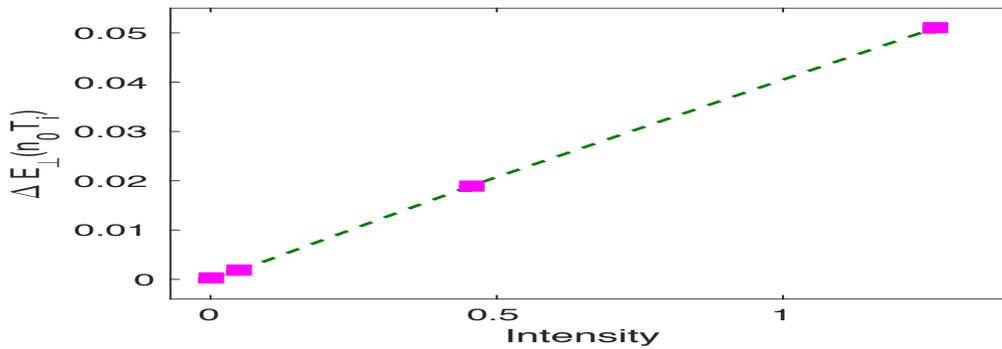
## PARAMETRIC DECAY INSTABILITY

In this section we have carried out the parametric decay instability of a pump wave  $(\omega_0, \vec{k}_0)$  into a lower frequency ion cyclotron quasi mode  $(\omega, \vec{k})$  and an ion Bernstein sideband  $(\omega_1, \vec{k}_1)$ , where  $\omega_0 = \omega_1 + \omega$  and  $\vec{k}_0 = \vec{k}_1 + \vec{k}$ . At higher amplitudes these modes are coupled and exchange momentum and energy with each other. Fig 3. shows that the ion heating takes place in the perpendicular direction due to nonlinear ion Landau damping.



**FIGURE 3.** Time history of change of ion perpendicular kinetic energy in linear (green) and nonlinear (red) simulation.

Figure 4 shows that the change in the ion perpendicular kinetic energy is linearly proportional to the intensity of the pump wave. In our simulation we change the frequency of the pump wave to get the frequency matching condition. The nonlinear Landau damping on ion is maximum when this phase matching condition is satisfied (cf. Fig. 5 red color).



**FIGURE 4.** Change of ion perpendicular kinetic energy as a function of intensity of pump wave.

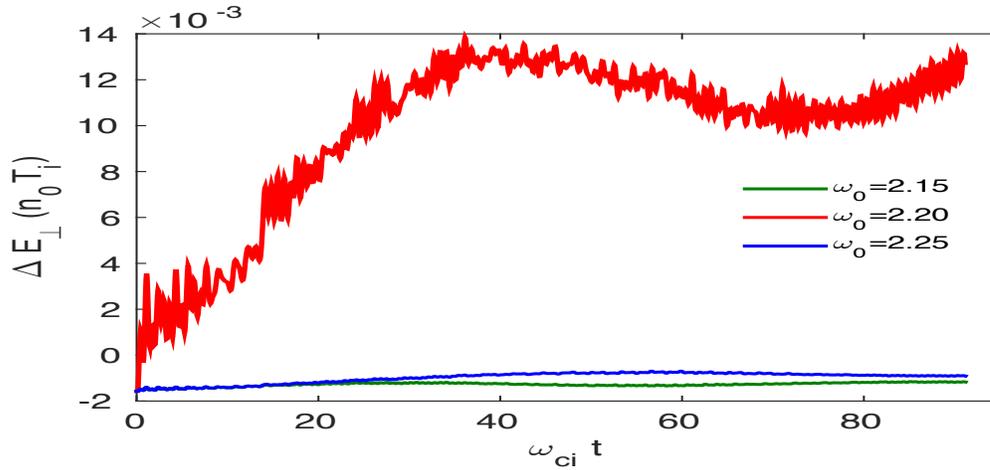


FIGURE 5. Time history of change of ion perpendicular kinetic energy for different pump wave frequencies.

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