ELECTROSTATIC ELECTRON FLUID-KINETIC HYBRID MODEL WITH EQUILIBRIUM RADIAL ELECTRIC FIELD

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1 Introduction

Recent progress in the use of resonant magnetic perturbations (RMP) in suppressing edge localized modes (ELMs) has placed increased emphasis on the role of the radial electric field (E_r) in turbulence suppression, since an increase of pedestal top turbulence and significant change in the E_r profile are observed during ELM suppression on the DIII-D tokamak. Hence, first principle, kinetic simulations that can capture the dynamics of E_r are crucial to elucidate the effects of RMP.

2 Gyrokinetic Equations in Toroidal Geometry

In the electrostatic and collisionless limit, the gyrokinetic equation for toroidal plasmas in an inhomogeneous magnetic field and five dimentional phase space with gyrocenter position $\mathbf{X}$, magnetic moment μ, and parallel velocity $v_\parallel$ is,

$$Lf_\alpha(\mathbf{X}, \mu, v_\parallel, t) \equiv \left[\frac{\partial}{\partial t} + (v_\parallel \mathbf{b}_0 + \mathbf{v}_d + \mathbf{v}_E) \cdot \nabla - \mathbf{b}_0^* \cdot (\mu \nabla B_0 + Z_\alpha \nabla \phi) \frac{\partial}{m_\alpha \partial v_\parallel}\right] f_\alpha = 0, \quad (1)$$

where,

$$\begin{aligned}
\phi &= \delta\phi + \phi_{ZF} \\
\mathbf{v}_E &= \frac{c\mathbf{b}_0 \times \nabla \phi}{B_0}, \\
\mathbf{v}_d &= \mathbf{v}_g + \mathbf{v}_c, \\
\mathbf{v}_g &= \frac{\mu}{m_\alpha \Omega_\alpha} \mathbf{b}_0 \times \nabla B_0, \\
\mathbf{v}_c &= \frac{v_\parallel^2}{\Omega_\alpha} \nabla \times \mathbf{b}_0, \\
\mathbf{b}_0^* &= \mathbf{b}_0 + \frac{v_\parallel}{\Omega_\alpha} \nabla \times \mathbf{b}_0, \\
\Omega_\alpha &= \frac{Z_\alpha B_0}{m_\alpha c}.
\end{aligned} \quad (2)$$

ϕ is the perturbed electrostatic potental and is separated into nonzonal and zonal components, $\delta\phi$ and ϕ_{ZF}, respectively, α represents the particle species, and m_α, Z_α, and $\mathbf{B}_0 = B_0\mathbf{b}_0$ are the particle mass, particle charge, and equilibrium magnetic field, respectively. Now, L can be separated into equilibrium, nonzonal, and zonal parts. Specifically,

$$L = L_0 + \delta L + L_{ZF} \tag{3}$$

where,

$$\begin{aligned} L_0 &= \frac{\partial}{\partial t} + (v_\parallel \mathbf{b}_0 + \mathbf{v}_d)\cdot\nabla - \mathbf{b}^*\cdot\mu\nabla B_0 \frac{\partial}{m_e\partial v_\parallel}, \\ \delta L &= \delta\mathbf{v}_E\cdot\nabla - \mathbf{b}^*\cdot Z_e\nabla\delta\phi\frac{\partial}{m_e\partial v_\parallel}, \\ L_{ZF} &= \frac{c\mathbf{b}_0\times\nabla\phi_{ZF}}{B_0}\cdot\nabla - \frac{v_\parallel}{\Omega_e}\nabla\times\mathbf{b}_0\cdot Z_e\nabla\phi_{ZF}\frac{\partial}{m_e\partial v_\parallel}, \end{aligned} \tag{4}$$

and, $\delta\mathbf{v}_E = \frac{c\mathbf{b}_0\times\nabla\delta\phi}{B_0}$.

3 Electron Kinetic Response

The gyrokinetic toroidal code (GTC) employs a pertubative scheme, meaning that the perturbed part of the particle distribution is evolved instead of the full distribution. This is done for the purpose of removing particle noise.

3.1 δf Scheme

The electron response can be separated into equilibrium and perturbed parts, $f_e = f_{e0} + \delta f_e$, with,

$$L_0 f_{e0} = 0, \tag{5}$$

defining the equilibrium distribution, the neoclassical solution. However, for simplicity, we will facilitate this work by using a local Maxwellian approximation, $f_{e0} \approx \frac{n_{e0}}{(2\pi T_e/m_e)^{3/2}}\exp\Big[-\frac{2\mu B_0 + m_e v_\parallel^2}{2T_e}\Big]$. After recasting f_e, we can rewrite Eq. 1 as,

$$\begin{aligned} L\delta f_e &= -Lf_{e0} \\ &= -(\delta L + L_{ZF})f_{e0}. \end{aligned} \tag{6}$$

Furthermore, the perturbed part of the distribution can be split into into a dominant adiabatic part, $\delta f_e^{(0)}$, and a smaller kinetic part, δh_e. Namely, $\delta f_e = \delta f_e^{(0)} + \delta h_e$, with $\delta f_e^{(0)} >> \delta h_e$.

To obtain an expression for $\delta f_e^{(0)}$, we expand Eq. 1 to first order in $\omega/k_\parallel v_\parallel$, assuming that $k_\perp L_p >> 1$, where L_p is the equilibrium pressure inhomogeneity scale length, obtaining:

$$\begin{aligned} & v_\parallel \mathbf{b}_0 \cdot \nabla \delta f_e^{(0)} - \mathbf{b}_0 \cdot Z_e \nabla \delta\phi \frac{\partial}{m_e \partial v_\parallel} f_{e0} = 0, \\ & \delta f_e^{(0)} = -\frac{Z_e \delta\phi}{T_e} f_{e0}. \end{aligned} \tag{7}$$

Neglecting terms of second order in pertubation, Eq. 5 can now be rewitten as,

$$L\delta h_e = -L(f_{e0} + \delta f_e^{(0)}) = -(\delta L + L_{ZF}) f_{e0} - f_{e0}(L_0 + L_{ZF}) \frac{\delta f_e^{(0)}}{f_{e0}}, \tag{8}$$

where,

$$\begin{aligned} \delta L f_{e0} &= \delta \mathbf{v}_E \cdot (\nabla f_{e0}|_{v_\perp} - \frac{\mu \nabla B_0}{T_e} f_{e0}) + v_\parallel (\mathbf{b}_0 + \frac{v_\parallel}{\Omega_e} \nabla \times \mathbf{b}_0) \cdot \frac{Z_e}{T_e} f_{e0} \nabla \delta\phi, \\ L_{ZF} f_{e0} &= \frac{c\mathbf{b}_0 \times \nabla \phi_{ZF}}{B_0} \cdot (\nabla f_{e0}|_{v_\perp} - \frac{\mu \nabla B_0}{T_e} f_{e0}) + \frac{v_\parallel^2}{\Omega_e} \nabla \times \mathbf{b}_0 \cdot \frac{Z_e}{T_e} f_{e0} \nabla \phi_{ZF}, \\ f_{e0} L_0 \frac{\delta f_e^{(0)}}{f_{e0}} &= \frac{\partial}{\partial t} \frac{\delta f_e^{(0)}}{f_{e0}} + (v_\parallel \mathbf{b}_0 + \mathbf{v}_d) \cdot \nabla \frac{\delta f_e^{(0)}}{f_{e0}}, \\ f_{e0} \delta L_{ZF} \frac{\delta f_e^{(0)}}{f_{e0}} &= \frac{c\mathbf{b}_0 \times \nabla \phi_{ZF}}{B_0} \cdot \nabla \frac{\delta f_e^{(0)}}{f_{e0}}. \end{aligned} \tag{9}$$

Here, $\nabla f|_{v_\perp}$ represents that $v_\perp$ is held constant, as opposed to μ, when taking the gradient. After simplifying, Eq. 8 becomes,

$$L\delta h_e = f_{e0} \left[\frac{\partial}{\partial t} \frac{Z_e \delta\phi}{T_e} - \delta \mathbf{v}_E \cdot \nabla ln f_{e0}|_{v_\perp} - (\mathbf{v}_d + \delta \mathbf{v}_E) \cdot \left(\frac{Z_e \phi_{ZF}}{T_e} \right) \right]. \tag{10}$$

4 Equilibrium Radial Electric Field

To add the effects of the radial electric field, the definition of the electrostatic potential is modified:

$$\phi = \delta\phi + \phi_{ZF} + \phi_{eq}, \tag{11}$$

where, ϕ_{eq} is the equilibrium potential. Then L_0 becomes,

$$L_0 = \frac{\partial}{\partial t} + (v_\parallel \mathbf{b}_0 + \mathbf{v}_d + \mathbf{v}_{E_{eq}}) \cdot \nabla - \mathbf{b}^* \cdot \mu \nabla B_0 \frac{\partial}{m_e \partial v_\parallel}, \tag{12}$$

where, the new term in the operator, $\mathbf{v}_{E_{eq}} = \frac{c\mathbf{b}_0 \times \nabla \phi_{eq}}{B_0}$, is in red. Eq. 10 now becomes,

$$L\delta h_e = f_{e0} \left[\frac{\partial}{\partial t} \frac{Z_e \delta\phi}{T_e} - \delta \mathbf{v}_E \cdot \nabla ln f_{e0}|_{v_\perp} + \mathbf{v}_{E_{eq}} \cdot \nabla \frac{Z_e \delta\phi}{T_e} - (\mathbf{v}_d + \delta \mathbf{v}_E) \cdot \nabla \left(\frac{Z_e \phi_{ZF}}{T_e} \right) \right]. \tag{13}$$

The new term in Eq. 13 can be combined with the last term yielding,

$$L\delta h_e = f_{e0}\left[\frac{\partial}{\partial t}\frac{Z_e\delta\phi}{T_e} - \delta\mathbf{v}_E\cdot\nabla ln f_{e0}|_{v_\perp} - \mathbf{v}_d\cdot\nabla\Big(\frac{Z_e\phi}{T_e}\Big) - \delta\mathbf{v}_E\cdot\nabla\Big(\frac{Z_e(\phi_{ZF}+\phi_{eq})}{T_e}\Big)\right]. \quad (14)$$

4.1 Magnetic Coordinates

In Boozer coordinates, with $\mathbf{B}_0 = I\nabla\theta + g\nabla\zeta$ the new term in Eq. 13 becomes,

$$\begin{aligned}
-\mathbf{v}_{E_0}\cdot\nabla\frac{\delta f_e^{(0)}}{f_{e0}} &= -\frac{c\mathbf{b}_0\times\nabla\phi_{eq}}{B_0}\cdot\Big(-\frac{Z_e\nabla\delta\phi}{T_e}\Big)\\
&= \frac{Z_e}{T_e}\frac{c}{B_0^2 J}\mathbf{B}_0\times\frac{\partial\phi_{eq}}{\partial\psi}\nabla\psi\cdot\nabla\delta\phi\\
&= \frac{Z_e}{T_e}\frac{c}{B_0^2 J}\frac{\partial\phi_{eq}}{\partial\psi}(g\mathbf{e}_\theta - I\mathbf{e}_\zeta)\cdot\left(\frac{\partial\delta\phi}{\partial\psi}\nabla\psi + \frac{\partial\delta\phi}{\partial\theta}\nabla\theta + \frac{\partial\delta\phi}{\partial\zeta}\nabla\zeta\right)\\
&= \frac{Z_e}{T_e}\frac{c}{B_0^2 J}\frac{\partial\phi_{eq}}{\partial\psi}\left(g\frac{\partial\delta\phi}{\partial\theta} - I\frac{\partial\delta\phi}{\partial\zeta}\right).
\end{aligned} \quad (15)$$