Electron drift-kinetic equation

High-order electron drift-kinetic equation

$$\frac{dw_e}{dt} = \left(1 + \frac{\delta f_e^{(0)}}{f_{0e}} + w_e\right)\left[\mathbf{v}_E \cdot \kappa - \frac{\partial}{\partial t}\frac{\delta f_e^{(0)}}{f_{0e}} - \mathbf{v}_d \cdot \nabla \frac{\delta f_e^{(0)}}{f_{0e}} + \frac{e}{T_e}\left(\mathbf{v}_g + \mathbf{v}_c\left(1 - \frac{v_{\|0}}{v_\|}\right)\right) \cdot \nabla\phi + \frac{1}{B_0}\mathbf{b}_0 \times \nabla \frac{\delta f_e^{(0)}}{f_{0e}} \cdot \nabla\langle\phi\rangle + \frac{v_\| - v_{\|0}}{T_e}\frac{\partial\langle A_\|\rangle}{\partial t} - \frac{e}{T_e}v_{\|0}E_\| - v_{\|0}\frac{\mu}{T_e}\mathbf{b}_0 \times \nabla\lambda \cdot \nabla B_0\right] \quad (1)$$

Effective potential is found from the adiabatic response

$$\frac{e\phi_{\rm eff}^{(0)}}{T_e} = \frac{\delta n_e}{n_0} + \kappa_n \delta\psi. \quad (2)$$

The lowest order solution

$$\frac{\delta f_e^{(0)}}{f_{0e}} = \frac{e\phi_{\rm eff}^{(0)}}{T_e} - \kappa\delta\psi$$

The time-derivative term

$$\text{wpara} \equiv -\frac{\partial}{\partial t}\frac{\delta f_e^{(0)}}{f_{0e}} = -\frac{1}{n_0}\frac{\partial \delta n_e}{\partial t} - (\kappa_n - \kappa)\frac{\partial\delta\psi}{\partial t} = -\frac{1}{n_0}\frac{\partial\delta n_e}{\partial t} + (\kappa_n - \kappa)\frac{1}{q}\frac{\partial\phi_{\rm ind}}{\partial\theta} \quad (3)$$

The drift terms

$$\text{wdrift} \equiv \frac{e}{T_e}\mathbf{v}_d \cdot \nabla\phi = -\frac{1}{e}\frac{1}{B_0^2 J}\left(\mu + \frac{m_e v_\|^2}{B_0}\right)\frac{e}{T_e}\left[g\left(\frac{\partial B_0}{\partial\psi}\frac{\partial\phi}{\partial\theta} - \frac{\partial B_0}{\partial\theta}\frac{\partial\phi}{\partial\psi}\right) - I\frac{\partial B_0}{\partial\psi}\frac{\partial\phi}{\partial\zeta}\right] \quad (4)$$

$$
\begin{aligned}
\text{wdriftind} \equiv -\mathbf{v}_d \cdot \nabla \frac{\delta f_e^{(0)}}{f_{0e}} = \\
\frac{1}{e}\frac{1}{B_0^2 J}\left(\mu + \frac{m_e v_\parallel^2}{B_0}\right)\frac{e}{T_e}\left[g\left(\frac{\partial B_0}{\partial \psi}\frac{\partial \phi_{\text{eff}}}{\partial \theta} - \frac{\partial B_0}{\partial \theta}\frac{\partial \phi_{\text{eff}}}{\partial \psi}\right) - I\frac{\partial B_0}{\partial \psi}\frac{\partial \phi_{\text{eff}}}{\partial \zeta}\right] - \\
\frac{\kappa}{e}\frac{1}{B_0^2 J}\left(\mu + \frac{m_e v_\parallel^2}{B_0}\right)\left[g\left(\frac{\partial B_0}{\partial \psi}\frac{\partial \delta\psi}{\partial \theta} - \frac{\partial B_0}{\partial \theta}\frac{\partial \delta\psi}{\partial \psi}\right) - I\frac{\partial B_0}{\partial \psi}\frac{\partial \delta\psi}{\partial \zeta}\right] \quad (5)
\end{aligned}
$$

Zonal ExB convection term

$$
\begin{aligned}
\text{wzonal} \equiv \frac{1}{B_0}\mathbf{b}_0 \times \nabla \frac{\delta f_e^{(0)}}{f_{0e}} \cdot \nabla\langle\phi\rangle = \\
\frac{1}{B_0^2 J}\left[I\left(\frac{e}{T_e}\frac{\partial \phi_{\text{eff}}}{\partial \zeta} - \kappa\frac{\partial \delta\psi}{\partial \zeta}\right) - g\left(\frac{e}{T_e}\frac{\partial \phi_{\text{eff}}}{\partial \theta} - \kappa\frac{\partial \delta\psi}{\partial \theta}\right)\right]\frac{\partial\langle\phi\rangle}{\partial \psi} \quad (6)
\end{aligned}
$$