Comparative studies of nonlinear ITG and ETG dynamics

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Abstract. It is shown that the ITG-ETG symmetry is nonlinearly broken due to the different response of respectively electrons and ions to Zonal Flows (ZF). ITG is dominated by ITG-ZF nonlinear interactions and turbulence spreading, resulting in size-scaling of the associated turbulent transport. Meanwhile, nonlinear toroidal mode coupling dominates ETG saturation and ETG-ZF interactions enter on the longest nonlinear time scale only. The ETG turbulent transport level is smaller than values of experimental relevance.

1. Introduction and Background

The present work demonstrates that the crucial difference in nonlinear dynamic behaviors between ion temperature gradient (ITG) instabilities and electron temperature gradient (ETG) modes stands in the particle response to zonal flows (ZF). On one hand, for ITG, massless electron response imposes that the lowest order zonal density fluctuation vanishes; on the other hand, for ETG, the ion response is adiabatic, due to $k_\perp \rho_i \gg 1$, even for $n = m = 0$ [see Eq. (1) below]. The main consequence of these different behaviors is that the specular symmetry between ITG and ETG, which holds linearly, is nonlinearly broken. This results into two main facts, *i.e.*, that the ZF polarizability is much lower for ITG [1] than for ETG [2,3] and that the dominant Drift Wave - Zonal Flow (DW-ZF) interaction is due to $\mathbf{E} \times \mathbf{B}$ nonlinearity for ITG, while ETG are characterized by the usual Hasegawa-Mima nonlinear coupling [4] (Section II). Based on these facts, the paradigm model for ITG, presented here, assumes that different DW interactions on the shortest non-linear time scale are mediated by ZF, which is spontaneously generated by ITG via modulational instability [5]. The resulting coherent model demonstrates turbulence spreading [6–11] to be the cause of transport scaling with system size [6–10] (Section III). The non-linear saturated state can be either coherent, with limit cycles, or chaotic, depending on proximity to marginal stability [9,10]. Meanwhile, ZF spontaneous generation is the weakest nonlinear dynamics for ETG, which saturate via nonlinear toroidal couplings that transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths [2,3] (Section IV). The ETG turbulence is dominated by nonlinearly generated radial streamers, but both fluctuation intensity and transport level are independent of the streamer size [2,3]. Nonlinear Gyrokinetic particle simulations indicate that typical ETG transports are smaller than those of experimental relevance [2,3].
In the following, we assume electrostatic DWs, whose linear mode structures are represented in the ballooning space [12] as

$$\delta\phi = e^{in\zeta} A(r) \sum_m e^{-im\vartheta} \int_{-\infty}^{\infty} e^{-i(nq-m)\theta} \Phi(\theta; r) d\theta \ \ , \tag{1}$$

where ζ is the ignorable periodic (toroidal) angle and, for convenience, we assume field-aligned flux coordinates (r, ϑ, ζ), with r the radial (flux) variable, ϑ the poloidal angle and $q(r) \equiv \mathbf{B} \cdot \nabla\zeta / \mathbf{B} \cdot \nabla\vartheta$. The mode *radial envelope* $A(r)$ is characterized by a spatial scale

L_A, such that $L_p \gg L_A \gg 1/|nq'|$, $1/|nq'|$ being the characteristic radial scale length of the single poloidal harmonics and L_p the plasma inhomogeneity scale length. Furthermore, $\Phi(\theta; r)$ describes the *parallel mode structure* along the extended poloidal angle θ [12], while translational invariance is implicit in Eq. (1) when considering that r dependencies in $\Phi(\theta; r)$ reflect slow radial equilibrium variations on the scale L_p only. Consistently with Eq. (1), the linear DW mode structures can be described with three degrees of freedom [2,3]: the toroidal mode number n, the parallel mode structure reflecting the radial width of a single poloidal harmonic m, and radial mode envelope $A(r) = \exp i \int \theta_k d(nq)$. Correspondingly, nonlinear interactions can take the following three forms: mode coupling between two n's, distortion of the parallel mode structure, and modulation of the radial envelope. Radial envelope modulation via generation of zonal flows dominates in ITG turbulence [5]. ETG turbulence, meanwhile, is regulated by nonlinear toroidal mode couplings [2,3].

2. Dynamics of Drift Wave - Zonal Flow interactions: ITG - ETG broken symmetry

The non-linear equations describing the dynamics of DW-ZF interactions can be systematically derived from the non-linear gyrokinetic equation [13]:

$$\begin{aligned}
&\left(\partial_t + v_\parallel \partial_\parallel + i\omega_d\right)_k \overline{\delta H}_k = \mathrm{i}\,(e/m)\, QF_0 J_0(\gamma)\delta\phi_k - (c/B)\,\mathbf{b}\cdot\left(\mathbf{k}''_\perp \times \mathbf{k}'_\perp\right) J_0(\gamma')\delta\phi_{k'}\overline{\delta H}_{k''} \quad , \\
&QF_0 = \omega_k \left(\partial F_0/\partial v^2/2\right) + (\mathbf{k}/\omega_c)\cdot \mathbf{b}\times\nabla F_0 \quad , \qquad (2)
\end{aligned}$$

where, as usual, the fluctuating particle distribution function has been separated into adiabatic and non-adiabatic response, $\overline{\delta H}$:

$$\delta F = \frac{e}{m}\delta\phi \frac{\partial}{\partial v^2/2} F_0 + \sum_{\mathbf{k}_\perp} \exp\left(-\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{v}\times\mathbf{b}/\omega_c\right)\overline{\delta H}_k \quad . \qquad (3)$$

In Eqs. (2) and (3), $\partial_\parallel = \mathbf{b}\cdot\nabla$, $\mathbf{b} = \mathbf{B}/B$, ω_d is the magnetic drift frequency, F_0 is the particle equilibrium distribution function, $\gamma \equiv k_\perp v_\perp/\omega_c$, ω_c is the cyclotron frequency, J_0 is the Bessel function of zero order, $\mathbf{k} = \mathbf{k}' + \mathbf{k}''$ and the other notation is standard. In Eq. (2), the linear response $\propto QF_0$ and the $\mathbf{E}\times\mathbf{B}$ nonlinearity are readily recognized on the right hand side (RHS). Assuming a plasma equilibrium with one ion species and density $n_i = n_e = n_0$, the non-linear equations for the DW-ZF system are finally obtained in the form of the quasi-neutrality conditions

$$n_0 e^2 \left(1/T_i + 1/T_e\right)\delta\phi_k = \left\langle eJ_0(\gamma_i)\overline{\delta H}_i\right\rangle_k - \left\langle eJ_0(\gamma_e)\overline{\delta H}_e\right\rangle_k \quad . \qquad (4)$$

Further progress with the $n \neq 0$ Eqs. (4) can be made by formally separating linear from non-linear particle responses as $\overline{\delta H} \equiv \overline{\delta H}^L + \overline{\delta H}^{NL}$ and explicitly solving for either $\overline{\delta H}_i^{NL}$ or $\overline{\delta H}_e^{NL}$ in the fluid limit for the ITG or ETG cases, respectively. In this way we obtain:

$$\begin{aligned}
&\left(n_0 e^2/T_i\right)\left(1 + T_i/T_e\right)\delta\phi_k - \left\langle eJ_0(\gamma_i)\overline{\delta H}_i^L\right\rangle_k + \left\langle eJ_0(\gamma_e)\overline{\delta H}_e^L\right\rangle_k = \\
&\quad -(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot\left(\mathbf{k}''_\perp\times\mathbf{k}'_\perp\right)\delta\phi_{k'} J_0(\gamma''_e)\overline{\delta H}_{ek''}\right\rangle_k - \left\langle eJ_0(\gamma_e)\overline{\delta H}_e^{NL}\right\rangle_k \\
&\quad -(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot\left(\mathbf{k}''_\perp\times\mathbf{k}'_\perp\right)\left[J_0(\gamma_i)J_0(\gamma'_i) - J_0(\gamma''_i)\right]\delta\phi_{k'}\overline{\delta H}_{ik''}\right\rangle_k \quad , \qquad (5)
\end{aligned}$$

for the ITG case [5,8,9], where all non-linear couplings have been isolated on the RHS. Meanwhile, following the same procedure as that described in Ref. [9], for ETG we obtain:

$$\begin{aligned}\left(n_0 e^2/T_e\right)(1+T_e/T_i)\,\delta\phi_k - \left\langle eJ_0(\gamma_i)\overline{\delta H}_i^L\right\rangle_k + \left\langle eJ_0(\gamma_e)\overline{\delta H}_e^L\right\rangle_k = \\ +(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)\,\delta\phi_{k'}J_0(\gamma''_i)\overline{\delta H}_{ik''}\right\rangle_k + \left\langle eJ_0(\gamma_i)\overline{\delta H}_i^{NL}\right\rangle_k \\ +(i/\omega_k)\left\langle (ec/B)\,\mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)\,[J_0(\gamma_e)J_0(\gamma'_e)-J_0(\gamma''_e)]\,\delta\phi_{k'}\overline{\delta H}_{ek''}\right\rangle_k \quad . \end{aligned} \tag{6}$$

Equations (4) to (6) clearly reflect the well-known symmetry of ITG and ETG dynamics when electron and ions are exchanged. However, while this symmetry is well preserved in the linear limit, it is broken nonlinearly due to the different ZF response of electrons and ions, respectively. In fact, the $n \neq 0$ electron response to ITG is (*nearly*) adiabatic, due to their vanishingly small inertia. Similarly, $n \neq 0$ ion response to ETG is (*nearly*) adiabatic as well, this time because of their large Larmor orbit compared to the perpendicular wavelength, *i.e.* $k_\perp\rho_i \propto (m_i/m_e)^{1/2} \gg 1$. For the same reasons, the $n \neq 0$ nonlinear electron and ion responses, respectively, can be neglected for ITG and ETG. In this way, and assuming $\gamma_i \ll 1$ for ITG and $\gamma_e \ll 1$ for ETG, only the last nonlinear term survives in Eqs. (5) and (6), *i.e.* the Hasegawa-Mima nonlinearity [4], along with the $\mathbf{E}\times\mathbf{B}$ nonlinearity terms $\propto \langle \mathbf{b}\cdot(\mathbf{k}''_\perp\times\mathbf{k}'_\perp)\,\delta\phi_{k'}J_0(\gamma''_{e,i})\overline{\delta H}_{e,ik''}\rangle$, with $\mathbf{k}'' = \mathbf{e}_r k_z = -i\mathbf{e}_r\partial_r$, k_z being the ZF wave vector. The modulation interaction model [5] considers a coherent linearly unstable DW (pump), interacting with collisionally damped ZF [1] in the presence of damped sidebands due to DW-ZF nonlinear interactions. In the case of ITG, the density perturbation caused by ZF is identically zero, due to massless electron response. Thus, $\overline{\delta H}_{ez} = -(e/T_e)F_{0e}\delta\phi_z$ and Eq. (5) is dominated by the $\mathbf{E}\times\mathbf{B}$ nonlinearity for $\gamma_i \ll 1$. In the local limit [5], Eq. (5) gives

$$A_+ = -i\,(c/B)\,(\omega_0\partial_{\omega_0}D_{Ri})^{-1}\,(\omega_z+\Delta+i\gamma_d)^{-1}\,(T_i/T_e)\,k_\theta k_z A_0 A_z \tag{7}$$

for the A_+ sideband amplitude, with ω_0 the ITG real frequency, D_{Ri} the Hermitian part of the ITG dielectric function, ω_z the ZF frequency, Δ the ITG frequency shift due to finite radial envelope width, γ_d the sideband damping rate, and A_0 and A_z the ITG and ZF amplitudes. The complex conjugate sideband amplitude A_- satisfies $A_- = A_+^*$. Furthermore, D_{Ri} is computed with an integral over the ballooning space [9]

$$D_{Ri} = \int_{-\infty}^{+\infty}\Phi_0\left[(1+T_i/T_e)\,\Phi_0 - (T_i/n_0 e)\left\langle J_0(\gamma_i)\overline{\delta H}_i^L\right\rangle/A_0\right]d\theta \quad , \tag{8}$$

having chosen to normalize the ballooning linear eigenfunction Φ_0 of the *pump* ITG such that $\int_{-\infty}^{+\infty}\Phi_0^2 d\theta = 1$. Meanwhile, rewriting Eq. (5) for the $n = 0$ ZF component, and defining the ZF collisional dissipation as $\gamma_z \approx (1.5\epsilon\nu_{ii})^{-1}$ [14], $\epsilon = r/R_0$, it is readily shown that [5,8,9]

$$(\omega_z + i\gamma_z)\,\chi_{iz}A_z = -i\,(c/B)\,k_\theta k_z\alpha_i k_z^2\rho_i^2\,(A_+A_0^* - A_0A_-) \quad , \tag{9}$$

where $\alpha_i = \delta P_{\perp i}/(en_0\delta\phi) + 1$ [5] and $\chi_{iz} \simeq 1.6q^2\epsilon^{-1/2}k_z^2\rho_i^2$ is the ITG ZF polarizability [1]. With Eqs. (7) and (9) and assuming $\omega_z = i\Gamma_z$, the ZF modulational instability growth rate [5] is readily shown to scale linearly with $|A_0|$ well above its onset threshold, *i.e.* $\Gamma_z = \gamma_M$ with

$$\gamma_M^2 = \left(\frac{c}{B}k_\theta k_z\right)^2\frac{(T_i/T_e)}{\omega_0\partial_{\omega_0}D_{Ri}}\,\frac{k_z^2\rho_i^2\alpha_i}{\chi_{iz}}|A_0|^2 \quad . \tag{10}$$

In the case of ETG, the ion response to ZF is still adiabatic due to $\gamma_i \gg 1$. Thus, there is a finite density perturbation caused by ZF, $\overline{\delta H}_{iz} = 0$ and Eq. (6) is dominated by the Hasegawa-Mima nonlinearity ($\gamma_e \ll 1$) [4]. In the local limit [5], Eq. (6) gives

$$A_+ = -i\,(c/B)\,(\omega_0 \partial_{\omega_0} D_{Re})^{-1}\,(\omega_z + \Delta + i\gamma_d)^{-1}\,\alpha_e \left(\left\langle\left\langle k_\perp^2 \right\rangle\right\rangle - k_z^2\right) \rho_e^2 k_\theta k_z A_0 A_z \quad , \qquad (11)$$

where $\alpha_e = \delta P_{\perp e}/(en_0\delta\phi) - 1$ [2,3] and $\langle\langle k_\perp^2 \rangle\rangle = \int_{-\infty}^{+\infty} \Phi_0^2 k_\perp^2 d\theta$. Meanwhile, D_{Re} is defined as [2,3] $D_{Re} = \left\langle\left\langle \left[(1 + T_e/T_i) + (T_e/n_0 e)\left\langle J_0(\gamma_e)\overline{\delta H}_e^L \right\rangle/\delta\phi\right]\right\rangle\right\rangle$ in analogy with Eq. (8). Note that, because of the difference in ZF response and the vanishing of $\mathbf{E}\times\mathbf{B}$ nonlinearities, Eq. (11) gives a $\approx \langle\langle k_\perp^2 \rangle\rangle\,\rho_e^2$ weaker sideband excitation for ETG than Eq. (7) for ITG. Similarly, ZF excitation rate is also weaker for ETG than for ITG since the ZF ETG polarizability is $\chi_{ez} \simeq (T_e/T_i)$ [2,3], *i.e.* larger than χ_{iz}, and the ZF evolution equation in the ETG case is obtained from Eq. (9) by substituting $\chi_{iz} \rightarrow \chi_{ez}$, $\alpha_i \rightarrow \alpha_e$ and $\rho_i \rightarrow \rho_e$. Therefore, the ZF modulational instability growth rate [5] well above its onset threshold is $\Gamma_z = \gamma_M$ with

$$\gamma_M^2 = \left(\frac{c}{B}k_\theta k_z\right)^2 \frac{(T_i/T_e)}{\omega_0\partial_{\omega_0} D_{Re}} k_z^2 \rho_e^2 \alpha_e^2 \left(\left\langle\left\langle k_\perp^2 \right\rangle\right\rangle - k_z^2\right) \rho_e^2 |A_0|^2 \quad . \qquad (12)$$

A direct comparison of Eqs. (10) and (12) demonstrates that ETG-ZF dynamics occurs on a $\approx (\sqrt{k_z^2 \langle\langle k_\perp^2 \rangle\rangle}\rho_e^2)^{-1}$ longer (normalized) time scale with respect to that of ITG-ZF [2,3]. For this reason, ZF are fairly ineffective in altering ETG turbulence structures [2,3], while they are crucial for regulating ITG turbulence level and the associated transport [6,15].

3. ITG nonlinear dynamics and size scaling of turbulent transport

For ITG, the Hasegawa-Mima nonlinearity, *i.e.* the nonlinear coupling between different n's (Section I), is an order $O(k_\perp^2\rho_i^2)$ smaller than the $\mathbf{E}\times\mathbf{B}$ nonlinearity, dominated by ZF [5,8,9]. Meanwhile, nonlinear distortions of the parallel mode structure are subdominant as well since $\Phi(\theta; r)$ is formed on a $\approx \omega_0^{-1}$ time scale, *i.e.* much shorter than the that of nonlinear processes $\tau_{\rm NL}$. All nonlinear interactions reflect on the radial envelope only, for which one can systematically derive nonlinear equations under the assumption that there exists a hierarchy among nonlinear wave-wave interactions, where the $\tau_{\rm NL} \approx \gamma_L^{-1}$ is set by ITG-ZF interactions [8,9]

$$\begin{aligned} \mathcal{L}_P P &= 2S\partial_x Z \\ \mathcal{L}_S S &= -P\partial_x Z \\ \mathcal{L}_Z Z &= 2\mathrm{Re}\left[P^*\partial_x S - S\partial_x P^*\right] \quad . \end{aligned} \qquad (13)$$

Here, P, S and Z stand for the suitably normalized [8,9] ITG, *sideband* (which appear as complex conjugate pairs) and ZF amplitudes, while the linear operators $\mathcal{L}_P, \mathcal{L}_S, \mathcal{L}_Z$, are defined as $\mathcal{L}_{P,S} = \partial_\tau - \bar{\gamma}_{P,S} - 2\delta^{1/2}\partial_x + i\Gamma(\lambda+\xi) + i\partial_x^2$ and $\mathcal{L}_Z = (\partial_\tau + \bar{\gamma}_z)$. Furthermore, we have extracted a $\propto \exp(-i\omega_0 t)$ time dependence from ITG and sideband envelopes, time is normalized as $\tau = |\gamma_{L\,P}(x = x_N)|t$, *i.e.*, to the "pump" ITG growth rate at a reference radial position x_N and $[\bar{\gamma}_{P,S}, \bar{\gamma}_z] = [\gamma_{L\,P,S}, \gamma_z]/|\gamma_{L\,P}(x = x_N)|$. The normalized radial coordinate x and the other quantities to be defined are given by $\Gamma = \omega_0/|\gamma_{L\,P}(x = x_N)|$, $\delta^{1/2} = (\xi\Gamma^{1/2})/(2\lambda^{1/2})$, $\xi = (\theta_{k0}\partial D_R/\partial\theta_{k0} - \theta_{k0}^2\partial^2 D_R/\partial\theta_{k0}^2)/(\omega_0\partial D_R/\partial\omega_0)$, $\lambda = (\theta_{k0}^2/2)(\partial^2 D_R/\partial\theta_{k0}^2)/(\omega_0\partial D_R/\partial\omega_0)$, $\partial_x = (\lambda^{1/2}\Gamma^{1/2})/(\theta_{k0}n(dq/dr))\partial_r$, with the dispersion function $D = D_R + iD_I$, $\gamma_L \equiv -D_I/(\partial D_R/\partial\omega_0)$ and $\theta_{k0}(r)$ implicitly defined via $D_R(r, \omega_0, \theta_{k0}) = 0$ [8,9]. This form of *linear propagators* is consistent with the present Mode Structure Decomposition approach [16],

based on both time and spatial scale separation of the mode structures, and includes the wave dispersive properties of the radial envelope to the leading order, *i.e.*, finite group velocity and focusing/de-focusing of the wave packet, which reflect the crucial importance of equilibrium geometry (toroidal) on the ITG intensity propagation via their dependencies on $\theta_{k0}(r)$ [8,9].
As discussed in Section II, ZFs are generated by ITG turbulence via modulational instability [5] and, in turn, act both as nonlinear damping as well as anti-potential well on the ITG pump [8,9]. This nonlinear interaction causes ITG to spread radially and eventually reach the linearly stable region [7–9,11,17]. In our model, turbulent transport is a local diffusive process due to the local turbulence level, that may eventually depend on the system size only via dependencies of the turbulence intensity on the global plasma equilibrium properties. For sufficiently strong growth rate, the mode grows at the local growth rate and nonlinearly saturates before any linear radial mode structure can form. The same happens for a sufficiently large system as well, when nonlinear interactions become important before the ITG traveling radial wave-packets sample regions of varying equilibrium, either because of the linear wave dispersive properties or of nonlinearly induced wave spreading. Under these circumstances, the system behaves as an infinite and uniform medium and turbulent transport is gyro-Bohm: in fact, fixed point solutions of Eqs. (13) for large $\bar{L}_p$ give [10]

$$I \simeq \frac{\bar{\gamma}_z(\bar{\gamma}_d + 2\bar{\gamma}_{P0})}{|\bar{\gamma}_d - \bar{\gamma}_{P0}|}\left(2 + \frac{2\Gamma^{1/2}}{5\bar{\gamma}_{P0}\bar{L}_p}\right) \quad , \tag{14}$$

for the turbulence intensity $I = \langle |P|^2 + 2|S|^2 \rangle$ dependence on the system size. Here, angular brackets denote spatial averaging, which we typically take to be $1/5$ of the linear unstable domain for the ITG pump P [8,9]. Furthermore, we have assumed a model $D_R = \omega/\omega_0 - 1 + \theta_k^2 + V(x)$, $V(x) = 1 - \exp(-x^2/\bar{L}_p^2)$, $\bar{L}_p = |ndq/dr|L_p/\Gamma^{1/2}$, $\bar{\gamma}_P = \bar{\gamma}_{P0} - (1 + \bar{\gamma}_{P0})V(x)$ and constantly damped sidebands, $\bar{\gamma}_S = -\bar{\gamma}_d$ [8–10]. In the opposite case, *i.e.* for either sufficiently small system or weak growth rate, the ITG traveling radial wave-packets sample regions of varying equilibrium and turbulent transport is Bohm-like: it can be shown that, for $\bar{L}_p\bar{\gamma}_{P0}\Gamma^{1/2} \approx 1$, the turbulence intensity is

$$I \simeq \frac{\bar{\gamma}_z\bar{\gamma}_d\bar{L}_p}{\sqrt{2\Gamma}}\left(1 + \frac{2}{\bar{\gamma}_d\Gamma^{1/2}\bar{L}_p}\right)^{-1}\left(1 + \frac{4\Gamma}{\bar{\gamma}_d^2\bar{L}_p^2}\right) \quad , \tag{15}$$

which scales with the system size. From the discussion above, it is not surprising that the control parameter from Bohm-like to gyro-Bohm transition is $\bar{L}_p\bar{\gamma}_{P0}\Gamma^{1/2}$, which is also the number of linearly unstable radial eigenmodes of the pump ITG [8–10].
Despite the coherence of the underlying nonlinear dynamics, the dynamic system of Eqs. (13) exhibits both fixed point and limit cycle attractors as well as chaotic behavior, depending on the system size and proximity to marginal stability [9,10]. However, even for the unstable fixed points, the turbulence intensity oscillates around the fixed point values, which provide a good estimate for the turbulence level and, thus, of turbulent transport [10]. Equations (14) and (15) provide an excellent fit to numerical solutions for the whole range of explored $\bar{L}_p$ and $\bar{\gamma}_{P0}$ [10].

4. ETG saturation via nonlinear toroidal coupling

The theoretical analysis of Section II demonstrates that nonlinear ETG-ZF dynamics is of negligible importance for ETG saturation and for setting the level of turbulent transport. The main reason is the ion (*nearly*) adiabatic response, which annihilates the $\mathbf{E} \times \mathbf{B}$ nonlinearity and makes the Hasegawa-Mima term the dominant interaction. The weak coupling between ETG and ZF is confirmed by *global* gyrokinetic simulations, which demonstrate the fairly ineffective role of ZFs in altering ETG turbulence structures [2,3,18,19]. However, there remain open discussions on the actual ETG saturation mechanism and on the corresponding level of turbulent transport [2,3,18–21].

Within our paradigm for treating nonlinear interactions (Section I) and given the analysis of Section II, ETG saturation can be set only by distortions of the parallel mode structures or by nonlinear coupling between different n's. This situation was recently studied by dedicated numerical simulations [2,3]. First of all, analyses of the nonlinear evolution of a single-n ETG, demonstrated that saturation occurs via the generation of an $(m = \pm 1, n = 0) = (m, n)^* \times (m \pm 1, n)$ mode, which broadens the radial width of poloidal harmonics, *i.e.*, increases $|k_\parallel|$ and enhances Landau damping. The $|k_\parallel|$ increase corresponds to a strengthening of the ballooning character of $\Phi(\theta; r)$ due to the modification of the θ-space potential well by the $(\pm 1, 0)$ mode. The elongated ETG eddies at saturation (*streamers*) are not appreciably altered with respect to the linear growth phase, suggesting weak ZF effects on ETG and excluding the mode saturation via excitation of a slab-like secondary Kelvin-Helmholtz (KH) instability [20,21]. Similar analyses, carried out with multiple-n ETG, show a much lower saturation level than in the single-n case, indicating that the nonlinear coupling between two different n's is the dominant process in the ETG saturation. It is readily recognized that this coupling is a truly toroidal process, since the Hasegawa-Mima term is $\propto \mathbf{b} \cdot \mathbf{k}'_\perp \times \mathbf{k}''_\perp$. It is the localized radial structure of the single poloidal harmonics on a $\approx 1/|nq'|$ scale that makes the coupling of two elongated streamers with $\mathbf{k}'_r, \mathbf{k}''_r \simeq 0$ possible, which otherwise could not effectively interact. Efficient nonlinear coupling between two different n's, say n_0 and n_1, imposes that poloidal harmonics be localized near the same radial position, *i.e.* that we consider a low order rational surface r_s, where $m_0/n_0 \simeq m_1/n_1 \simeq q(r_s) \equiv m_l/n_l$. Assuming the typical ETG unstable spectrum, an optimal ordering is found for $n_l \approx n_0^{1/2}$. Furthermore, we take $n_l = n_0 - n_1$ to satisfy mode number matching conditions. The low-n beat waves are *quasi modes* since they do not satisfy three-wave resonant conditions due to $1 > (\gamma_{L0}/\omega_0) \sim (\gamma_{L1}/\omega_1) \sim k_\perp \rho_e \sim n_0^{-1/4} > |\omega_0 - \omega_1|/\omega_0 \sim n_0^{-1/2}$ [2,3]. Due to toroidal geometry and their nonlinear nature, quasi modes are characterized by highly localized radial structures as well as long parallel wavelength, $k_\parallel \sim 1/(n_0^{1/2} q R_0)$ (not ballooning). Starting from the quasineutrality conditions for n_0, n_1, n_l modes in the form of Eq. (6), it is possible to systematically calculate the parallel mode structure of the quasi modes, and then derive the evolution equations for the normalized amplitudes $a_0(t)$, $a_1(t)$ and $a_l(t)$, where $a = eA/T_e$ and A is the local mode amplitude defined by Eq. (1). Given s the magnetic shear, we have [2,3]:

$$(\partial_t - \gamma_{L0}) a_0 = -\gamma_{NL1} a_1 a_l \ , \qquad (\partial_t - \gamma_{L1}) a_1 = \gamma_{NL0} a_0 a_l^* \ , \qquad \partial_t a_l = \gamma_{NLl} a_0 a_1^* \ , \quad (16)$$

where $\gamma_{NL0,1} = (\left\langle\left\langle k^2_{\perp 0,1} \right\rangle\right\rangle / W_l^2) k^2_{\theta 0,1} s \alpha_e |\omega_{ce}| (T_i/T_e) \rho_e^4$, $\gamma_{NLl} = (2n_l/n_0) k_\theta^4 s \alpha_e |\omega_{ce}| (T_i/T_e) \rho_e^4$ and, defining $\theta_\ell \equiv \theta + 2\pi\ell$,

$$\frac{\langle\langle k_\perp^2 \rangle\rangle}{k_\theta^2 W_l^2} = 4\pi^2 \sum_\ell \ell^2 e^{2\pi i \ell n q} \int_{-\infty}^{\infty} \left[4\pi^2 \ell^2 - \left(1 + s^2 \left\langle\left\langle \theta^2 \right\rangle\right\rangle \right) \right] \left[1 + s^2 \theta_\ell^2 \right] \Phi^*(\theta) \Phi(\theta_\ell) d\theta \ . \quad (17)$$

Note that, from Eqs. (16), the spectral transfer is toward longer poloidal wavelengths. Since $n_0 \sim n_1 \gg n_l$, we can generalize Eqs. (16) to the multiple n case in the continuum limit.

Introducing $I_n = |a_n|^2/2$ and $v_n = -\gamma_{NL} n_l |a_l|$, we finally have

$$(\partial_t - 2\gamma_{Ln}) I_n + v_n \partial_n I_n = 0 \ , \qquad (\partial_t + \gamma_l)|a_l| = 4\alpha_e |\omega_{ce}| (T_i/T_e) \rho_e^4 q' \int k_{\theta n}^3 I_n dn \ , \quad (18)$$

where we have introduced γ_l as the damping rate of the forced n_l-mode via $k_\| v_\|$ Landau damping. Since $v_n < 0$, Eqs. (18) indicate that ETG energy is gradually transferred to longer poloidal wavelengths via scattering off the low-n quasi modes, till saturation takes place due to enhanced damping and/or decreased drive. The corresponding turbulent transport level is smaller than values of experimental relevance, as demonstrated in global gyrokinetic simulations [2,3]. The crucial role of low-n quasi modes as *mediators* of the nonlocal spectral energy transfer [2,3] makes it necessary to properly treat the dynamics of these low mode numbers, which are characterized by highly localized radial structures and are very extended along the field lines, $k_\| \sim 1/(n_0^{1/2} q R_0)$. Underestimating the quasi mode amplitude or occupation number implies underestimating v_n, resulting in a larger ETG saturation level and turbulent transport. This point could help resolving the discrepancy between flux tube and global gyrokinetic particle simulation results [2,3,18–21].

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