Comment on "Electrostatic and Magnetic Transport of Energetic Ions in Turbulent Plasmas"

In a recent Letter [1], Hauff *et al.* state [see also Eq. (3) therein] without proof that the condition for the validity of orbit averaging is that the orbital time of the equilibrium periodic motion is shorter than the time for the perpendicular drift across a turbulence eddy by either the turbulence $E \times B$ drift or the equilibrium magnetic drift. The authors further state that the equilibrium magnetic drift dominates over the perturbed $E \times B$ drift in the high energy limit and that the orbit averaging is invalid since the perpendicular magnetic drift time is shorter than the equilibrium orbital time. This time-scale separation between the equilibrium drift across the eddy and the equilibrium periodic motion is identical to the spatial-scale separation between the turbulence eddy size (λ_c) and the guiding center orbit size (Δr), as is plainly expressed in Eq. (4) of Ref. [1], which is independent of the turbulence intensity.

This claim of a spatial-scale separation between equilibrium and perturbed fields is of both fundamental and practical significance. Fundamentally, the claim implies that the orbit-averaged theory is valid only if $\lambda_c > \Delta r$. This claim contradicts the textbook [2–4] notion that the time-scale separation between equilibrium and perturbed motions is the only requirement for the validity of the well-established orbit-averaged theory [5–7] in plasma physics. Practically, the requirement of the spatial-scale separation leads to an energy scaling in Ref. [1] different from that of the orbit-averaged theory [6–8] for the turbulent transport of energetic trapped particles.

In this Comment, we present a canonical perturbation theory to demonstrate that orbit averaging is valid for arbitrary radial eddy size. The orbit-averaged theory (averaging over the canonical angle variable) strictly follows from the existence of an adiabatic invariant (canonical action variable). The gyrokinetic quasilinear theory with orbit averaging for low-frequency electrostatic turbulence in an axisymmetric system has been rigorously derived [6]. Here we apply the general theory of Ref. [6] to the diffusivity for deeply trapped particles in a tokamak:

$$D_t \propto |k_\theta \phi_n J_0(k_\perp \rho_E) J_l(k_\perp \rho_d)|^2 \delta(\omega - n\bar{\omega}_p - l\omega_b). \quad (1)$$

The only assumption made in deriving this equation is equilibrium time-scale separations, i.e., $\Omega \gg \omega_b \gg \omega_p$, along with the usual quasilinear assumptions. Here, Ω is the particle cyclotron frequency, ω_b is the transit or bounce frequency of the guiding center parallel motion, and ω_p is the toroidal precessional drift frequency of the guiding center perpendicular motion (the overbar represents guiding center orbit averaging on the $1/\omega_b$ fast time scale). Moreover, ω is the perturbation mode frequency, ϕ_n is the electrostatic potential with a toroidal mode number n, k_θ and $k_\perp$ are the poloidal and perpendicular wave vector, respectively, and l is the bounce harmonics. The Bessel function $J_0^2(k_\perp \rho_E)$ comes from the width of the gyroradius and $J_l^2(k_\perp \rho_d)$ from the width of the guiding center drift orbit.

With an additional assumption of a separation between equilibrium and fluctuation time scales $\omega_b \gg \omega$, Eq. (1) becomes the guiding center bounce-averaged theory. Here, ω represents the maximum of perturbation mode frequency, inverse of turbulence autocorrelation time, and nonlinear frequency $\delta v/\lambda_c$ (δv is the perturbed $E \times B$ drift). The condition of $\omega_b \gg \omega$ is always satisfied for energetic particles, since ω is independent of the particle energy E for a given turbulence intensity and spectrum but ω_b is proportional to $E^{1/2}$. We now show that the orbit-averaging theory for deeply trapped energetic particles is valid for arbitrary radial eddy size. For low-n modes (large poloidal eddy size) but with arbitrary radial eddy size, the dominant term in Eq. (1) is $l = 0$. Equation (1) thus becomes $D_t \propto \sum_{n,l} |k_\theta \phi_n J_0(k_\perp \rho_E) J_l(k_\perp \rho_d)|^2 \delta(\omega - n\bar{\omega}_p)$, which is simply the usual bounce-averaged theory [5] also applicable to trapped-ion mode and fishbone oscillation [9]. The guiding center orbit-averaged potential $\bar{\phi} = \phi_n J_0(k_\perp \rho_d)$ is valid for arbitrary radial eddy size since $J_\parallel$ is conserved.

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