Nonlinear toroidal coupling: a new paradigm for plasma turbulence

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Abstract

Global gyrokinetic particle simulations find that electron temperature gradient (ETG) instability saturates via nonlinear toroidal couplings, which transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths. The electrostatic ETG turbulence is dominated by nonlinearly generated radial streamers. The length of streamers scales with the device size and is much longer than the distance between mode rational surfaces or electron radial excursions. Both fluctuation intensity and transport level are independent of the streamer size. The nonlinear toroidal couplings represent a new paradigm for the spectral cascade in plasma turbulence.

1 Introduction

Electron temperature gradients in magnetically confined plasmas provide expansion free energy for driving various drift-wave instabilities [1], which may induce high level electron heat transport often observed in toroidal experiments. Identifying the candidate instabilities and understanding the nonlinear interactions are the first step toward modeling and controlling the electron transport in fusion plasmas. Linear properties of the toroidal ETG instability [2] are well understood. The gyroBohm level of the ETG electron heat conductivity χ_e^{GB} from a heuristic mixing length estimate [3] is smaller than that of the ITG ion transport χ_i^{GB} by a factor of the square-root of ion-electron mass ratio, i.e., $\chi_e^{GB} \sim 1/60\chi_i^{GB}$ for deuterium plasmas. Since experimental measurements find, typically, $\chi_e, \chi_i \sim \chi_i^{GB}$, the ETG instability has generally been discarded as a potential driver for the anomalous electron transport. However, the nonlinear evolution of ETG and ITG could be very different. Whereas a $\mathbf{E} \times \mathbf{B}$ nonlinearity associated with zonal flows [4, 5, 6] dominates in the ITG turbulence, ETG turbulence is regulated by a much weaker polarization nonlinearity [7].

The renewed interest in the electrostatic ETG instability comes from gyrokinetic continuum (Vlasov) simulations using the flux-tube geometry [8], which found that elongated turbulence eddies, or radial streamers, drive a transport level of experimental relevance. However, in these simulations the scale length of ETG streamers is comparable to the simulation box size. This violates the fundamental assumption of the flux-tube simulation, which assumes that the radial correlation length of turbulence eddies is much shorter than the simulation box size and uses a periodic boundary condition in the radial direction. Meanwhile, radially nonlocal, global simulations using fluid models in a small tokamak [9] or a simplified equilibrium geometry [10] found that the ETG turbulent transport is smaller than the flux-tube simulation result by more than an order of magnitude, and concluded that ETG turbulence is unlikely responsible for the electron anomalous transport.

Key results from flux-tube simulations [8] of the ETG turbulence are that radially extended streamers form in the absence of strong zonal flows and that electron transport up to $60\chi_e^{GB}$ is driven by the electrostatic $\mathbf{E} \times \mathbf{B}$ convection. However, the direct relationship between the ETG streamer size and the electron transport, as well as the mechanism for the ETG saturation has not been established by direct numerical simulations or by first-principles theories. Furthermore, the substitution of the nonlinear decorrelation rate by the linear growth rate in the condition for the flow shear suppression [11] of ETG turbulence is questionable. We address these issues in our present studies utilizing a well-benchmarked, massively parallel, global gyrokinetic toroidal code (GTC) [4] to simulate the electrostatic ETG turbulence in a realistic tokamak.

In the present study, our global gyrokinetic particle simulations find that the ETG instability saturates via nonlinear toroidal couplings, which transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths. The electrostatic ETG turbulence is dominated by nonlinearly generated radial streamers, which have a nonlinear decorrelation rate much smaller than the linear growth rate. Both fluctuation intensity and transport level are independent of the streamer size, which scales with the device size and is much longer than the distance between mode rational surfaces or electron radial excursions.

The nonlinear toroidal couplings found in this study is a novel nonlinear interaction underlying the ETG poloidal spectral cascade. In this nonlinear mode coupling processes, two unstable high-n pump toroidal eigenmodes with toroidal mode numbers $n_0 \gg 1$ first drive a low-n quasi-mode with $n_l \sim n_0^{1/2}$. Next, the scattering of the pump modes on the quasi-mode creates secondary high-n eigenmodes with mode number $n_2 = n_0 - n_l$. This nonlinear process proceeds until all n-matching modes are populated, and results in a down-shift of the poloidal spectrum from linearly most unstable modes to nonlinearly dominant modes with longer poloidal wavelengths. This nonlocal interaction in the wavevector space is much like the Compton scattering with quasi-modes playing the role of quasi-particles. Three-mode resonant coupling is not operative due to the frequency mismatch. Moreover, similar poloidal spectral cascade occurs in the ITG/TEM turbulence [12]. Although zonal flows play a dominant role in saturating the ITG/TEM instability, the poloidal spectrum can not be determined [13] by interactions between ITG/TEM turbulence and zonal flows. Therefore, poloidal spectra of any toroidal drift wave turbulence are ultimately determined by drift wave-drift wave interactions. Recognizing this universal role, nonlinear toroidal couplings represent a new paradigm for plasma turbulence in toroidal geometry.

Our GTC simulation results have important implications on plasma turbulence studies. First, particle dynamics must be treated on the same footing as fluid nonlinearity. ETG radial streamers, which represent the $\mathbf{E} \times \mathbf{B}$ velocity field, are generated by nonlinear toroidal couplings. Linear wave-particle resonance can then be destroyed nonlinearly. Consequently, electron radial excursions are diffusive and much shorter than the streamers size, i.e., particles and fluid elements do not move together due to the parallel free streaming motion. While wave-wave couplings determine fluctuation characteristics, transport is driven by wave-particle interactions. This is a crucial difference between plasma turbulences and fluid turbulences, where fluid elements move together with the velocity field. Therefore, fluid concepts, such as mixing length rule, eddy turnover time, *etc*, do not correctly describe transport processes in plasma turbulences.

Secondly, toroidal geometry must be treated rigorously in studying toroidal drift wave turbulences. The nonlinear toroidal couplings are strictly geometry-specific effects. There is no such counterparts in the slab geometry, where two parallel streamers can not interact since the wavevectors satisfy $\mathbf{k_1} \times \mathbf{k_2} = 0$. All eigenmodes participate in nonlinear toroidal couplings, and therefore, the saturation amplitude may not be predicted accurately in nonlinear simulations using a small number of modes [14]. Furthermore, the radial variations of the safety factor q need to be retained in nonlinear simulations to properly account for the nonlinear wave-particle interactions. The parallel wavelength, which determines the resonant condition, varies over a radial scale length on the order of the distance between mode rational surfaces, which is only a few electron gyroradii in the ETG turbulence. Therefore, important kinetic effects may not be treated correctly when a uniform q is used in nonlinear flux-tube simulations even if the simulation box size is much shorter than the magnetic shear scale length. Note that q is treated as a constant in the linear simulation or local ballooning mode theory [15] because of the radial translational symmetry, which can be nonlinearly broken.

Finally, the contradictory results from ETG turbulence simulations between flux-tube codes and global codes are presumably consequences of differences in the respective geometry representations. While the toroidal geometry is treated rigorously in global codes, flux-tube codes make key approximations, the validity regime of which remains dubious for nonlinear simulations involving fluctuations with low toroidal mode numbers. Therefore, the flux-tube simulation is a reduced model, and its validity rests on the ability to recover results of more general global simulations in appropriate asymptotic regimes. This reasonable requirement has actually been at the center of recent debates arising from criticisms of GTC global simulation results. These criticisms are based on a practice of designing [14] global code results to "match those obtained with flux tubes".

2 ETG Saturation via Nonlinear Toroidal Coupling

The linear toroidal ETG eigenmodes can be described by three degrees of freedom: a toroidal eigenmode number n assuming axisymmetry, a parallel mode structure determined by the radial width of the poloidal mode number m, and a ballooning angle θ_0 representing the radial envelope of the linearly coupled m harmonics. Consequently, nonlinear interactions can take the following three forms: a nonlinear mode coupling between two n toroidal eigenmodes, a modification of the parallel mode structure, and a modulation of the radial envelope. The envelope modulation, i.e., the generation of zonal lfows, dominates in the ITG turbulence. In the ETG turbulence, all these interactions are formally on the same order. We study all these interactions and find that the coupling between two n eigenmodes, labeled as nonlinear toroidal coupling, is the dominant nonlinear interaction in the ETG turbulence. All simulations use a tokamak size $a = 1000\rho_e$ and diagnostics at a reference minor radius $r = 0.5a$ with a safety factor $q = 1.4$ and a magnetic shear $\hat{s} = 0.78$.

It has been suggested that ETG instability saturates when the linear growth of the primary ETG instability is balanced by a slab-like secondary Kelvin-Helmholtz (KH) instability [8]. In this process the linear streamer of a single toroidal eigenmode is broken up by the KH instability. To test this hypothesis, we first study the nonlinear saturation of a single toroidal eigenmode of $n_0 = 110$ with $k_\theta \rho_e = 0.31$. In this test case, we initially only allow the $n_0 = 110$ mode to grow from very small random noise, i.e., only the electric field associated with this mode is used in the calculation of particle orbits. The poloidal contour plot of density perturbation shows that the mode is dominated by a linear toroidal eigenmode with a ballooning angle $\theta_0 = 0$. The linear streamers are formed by linear toroidal couplings, where many poloidal m harmonics are linearly coupled in a single toroidal n eigenmode because of the magnetic field dependence on the poloidal angle. At $r = 0.5a$, the dominant m harmonic is $m_0 = qn_0 = 154$. When the amplitude of this mode is much higher than any other mode, all n modes are allowed to grow. Since the primary $n_0 = 110$ mode, the pump eigenmode, has the maximal linear growth rate, the amplitude of the pump continues to be much higher than other modes. After saturation of the $n = 110$ mode, the linear streamer is well preserved. Therefore, we do not find signature of the secondary KH instability. This is in contrast to the ITG case where zonal flows, generated through a modulational instability, break up linear ITG streamers [4].

At the saturation of the pump eigenmode, two most significant secondary modes at $r = 0.5a$ are that of $n = 0, m = \pm 1$, or $(0, 1)$ mode, and $n = 2n_0 = 220, m = 2m_0 \pm 1 = 307, 309$, or $(2n_0, 2m_0 \pm 1)$ mode. They are evidently generated by the following mode coupling process:

$$(n_0, m_0) + (n_0, m_0 \pm 1) \Rightarrow (0, \pm 1), (2n_0, 2m_0 \pm 1)$$

Each m harmonic peaks at the mode rational surface where $m = qn_0$ and decrease to very low amplitudes at neighboring mode rational surfaces for $m \pm 1$ harmonics. The radial width of m harmonics represents the parallel mode wavevector $k_{||}$. The wider radial width corresponds to the larger $k_{||}$. The radial profile of the m harmonics after nonlinear saturation clearly shown a widening of the m harmonics, i.e., an increase in $k_{||}$. Landau damping is then enhanced since the ETG linear frequency is larger than the transit frequency of thermal electrons. Therefore, the single-n ETG eigenmode saturates through the modification of the parallel mode structure. The coupling to the $(0,1)$ mode is of particular interest since all linearly unstable modes contribute to it in the fully nonlinear simulations discussed in the previous Section. The radial scale length of the $(0,1)$ mode is similar to the distance between mode rational surfaces of the high-n pump mode. Therefore, the assumption of an adiabatic ion response is valid even for this $n=0$ mode. Meanwhile, the zonal flow, or $(0,0)$ mode, is generated through modulation of the radial envelope. However, the amplitude of zonal flow is low and it does not breakup the linear ETG streamer.

We now study the nonlinear interaction between two n eigenmodes by adding more pump modes to the simulation. We find that the saturation amplitude of a single-n ETG mode is much higher than that in the presence of another eigenmode with a similar amplitude, suggesting that the nonlinear coupling between two eigenmodes is the dominant process in ETG saturation.

A conceptual difficulty we must first address regarding nonlinear mode coupling is whether two toroidal eigenmodes with $k_r = 0$ can nonlinearly interact. This is not so obvious since two streamers in a slab geometry with parallel wavevectors $\mathbf{k_1} \| \mathbf{k_2}$ would not nonlinearly interact due to the fact that $\mathbf{k_1} \times \mathbf{k_2} = 0$. This concept of slab streamers has perhaps misled the previous studies of toroidal drift wave turbulences, including both ITG and ETG. In fact, the situation is quite different for toroidal eigenmodes. Although toroidal streamers have an envelope $k_r = 0$, there is a "hidden" $k_r \sim \hat{s}k_\theta$ due to the localization of each m harmonics near the mode rational surface. Therefore, two toroidal eigenmodes can nonlinearly interact because of the unique ballooning mode structure. We note that this nonlinear mode coupling is strictly toroidal geometry specific since there is no slab counterpart of such streamer interactions. We thus call nonlinear couplings between two n modes as nonlinear toroidal couplings.

We now proceed to examine the nonlinear toroidal coupling of two n modes. In such a simulation, two toroidal eigenmodes, $n_0 = 110$ and $n_1 = 95$, are allowed to grow first, i.e., only these two pump waves feed back to the particle dynamics. When the amplitudes of these two modes are much higher than any other mode, all toroidal modes are allowed to grow, i.e., the electric field from all modes is used for calculating the particle motion.

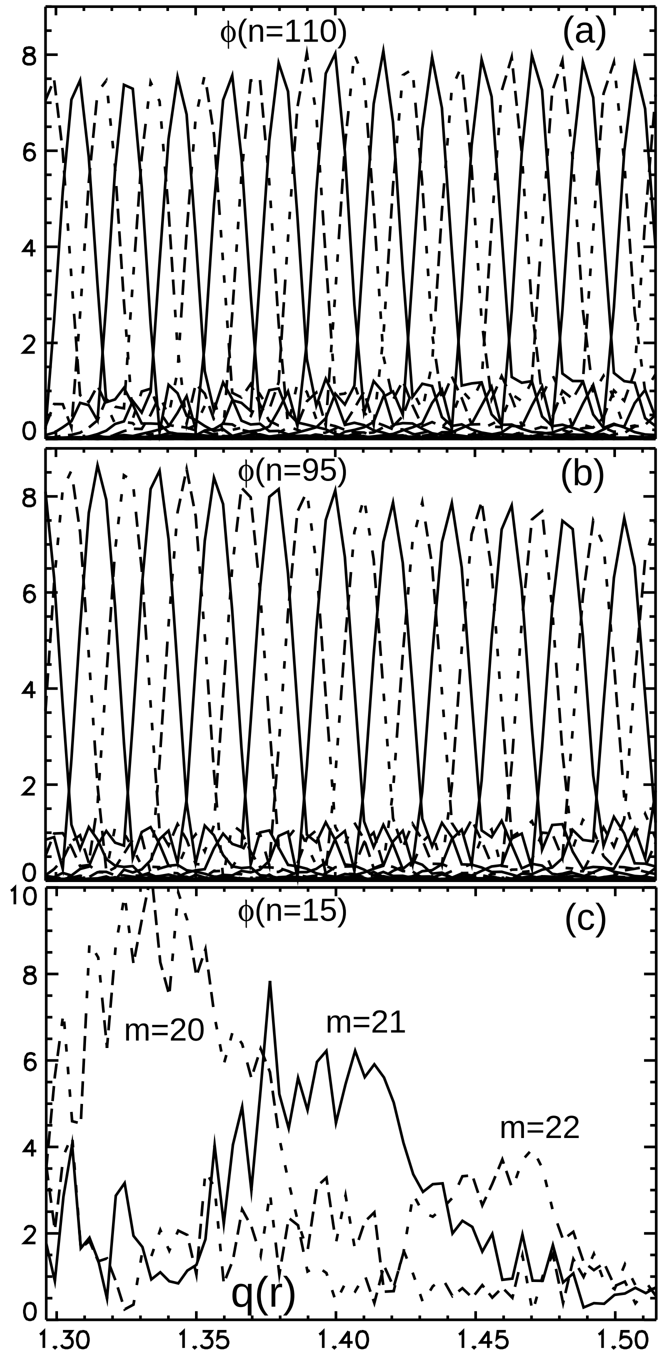

Figure 1: Radial profiles for the amplitude of (n, m) harmonics of pump toroidal eigenmodes $n = 110$ (Panel **a**, $e\phi/T_e \times 10^6$) and $n = 95$ (Panel **b**, $e\phi/T_e \times 10^6$) in linear phase, and of driven mode $n = 15$ (Panel **c**, $e\phi/T_e \times 10^5$) before saturation. Each solid or dashed line describes a (n, m) harmonics with $m = qn$ at mode rational surface. Safety factor $q(r)$ is used as the radial coordinate.

The radial profiles of these two eigenmode in linear phase are shown in the upper two panels of Fig. 1. Each m_0 harmonic of the n_0 mode, in addition to the coupling to the $m_0 \pm 1$ harmonics of the n_0 mode itself, interact most strongly with one m_1 harmonic of the n_1 mode, where m_0 and m_1 are the harmonics whose mode rational surfaces sit close to each other. The coupling proceeds as:

$$(n_0, m_0) + (n_1, m_1) \Rightarrow (n_0 \pm n_1, m_0 \pm m_1).$$

This coupling produces both a very high-n mode, $n_h = 205$, and a low-n mode, $n_l = 15$. The amplitude of the very high-n mode is much smaller since the coupling coefficients to the very high-n mode is much weaker than that to the low-n mode. This is because the intrinsic frequency, i.e., the inertia, of the very high-n mode is much higher than that of the low-n mode, and, furthermore, because the interacting wavevectors of the two pump eigenmodes are almost parallel in the coupling to the very high-n mode, whereas they are almost perpendicular in the coupling to the low-n mode. The low-n mode is a forced oscillation, i.e., a quasi-mode, since its intrinsic frequency is much smaller than the frequency difference between the two pump eigenmodes, which is on the order of their linear growth rates. Each m harmonic of the low-n quasi-mode is localized near its own mode rational surface, as shown in the lower panel of Fig. 1, which also shows that the radial scale length of the low-n quasi-mode is similar to the distance between the mode rational surfaces of the pump modes, which is the radial width of the interactions between m-harmonics. This is also confirmed in a poloidal contour plot of this quasi-mode, which shows that the radial eddy size is very small. Near mode rational surface, the low-n quasi-mode has a very long parallel wavelength, i.e., $k_{||} \sim 0$. Therefore, the low-n quasi-mode does not possess the ballooning mode structure.

The generation of the low-n quasi-mode, $n_l \equiv n_0 - n_1 = 15$, by the two high-$n$ pump modes, $n_0 = 110$ and $n_1 = 95$, is simply the first step of the nonlinear toroidal coupling. Once generated, the quasi-mode n_l couples back to the two pump modes and generates secondary n_2 modes, $n_1 - n_l = 80$, and $n_0 + n_l = 125$, as shown in the upper panel of Fig. 2 just before the saturation of the pump modes. In turn, each secondary n_2 mode couples with the far-side pump mode to generate another quasi-mode $n_l = 30$. Again, the coupling between the low-n quasi-mode $n_l = 30$ with the pump modes generates further secondary modes $n_2 = 65$, and 140. These successive coupling processes proceed until all n-modes that satisfy the n-matching condition are populated with either a quasi-mode or a secondary mode, as shown in the middle panel of Fig. 2 after the saturation of the pump modes. The amplitude of the higher-n secondary modes, $n_2 = 125, 140, \cdots$, is always smaller than the lower-n secondary modes, $n_2 = 80, 65, \cdots$. This indicates that the energy cascades preferably to lower-n secondary modes. Note that each coupling always involves a quasi-mode, a secondary ballooning mode, and a pump ballooning mode. The low-n quasi-modes do not contain much energy, nor do they drive much transport. Rather, they act as mediators that facilitates the transfer of energy from pump modes to secondary modes. Therefore, the nonlinear toroidal coupling can be viewed as a two-step process, first the generation of the low-n quasi-mode, and the subsequent energy transfer from the pump modes to the lower-n secondary modes. The second step is similar to the Compton Scattering [16] with the quasi-mode playing the role of the quasi-particle.

The parallel mode structure of the pump modes is also modified at saturation through coupling to the $(0, 1)$ harmonic (middle panel of Fig. 2). However, its amplitude decreases quickly due to Landau damping (lower panel of Fig. 2). The amplitude of the zonal flow, $(0, 0)$ mode shown in Fig. 2, is always very small in consistent with the fact that the envelope modulation is insignificant in this spectral cascade process. At the steady state, the ETG turbulence is dominated by nonlinearly-generated lower-n secondary mode streamers, which have longer intrinsic characteristic time scales and could be prone to the shearing effects of the equilibrium and zonal flows. Steady state is achieved both via modification by the $(0, 1)$ mode of the parallel mode structure of linearly unstable modes, which enhances Landau damping, and via energy transfer to low-n and high-n damped modes.

In summary, we find that the toroidal ETG instability saturates via nonlinear toroidal couplings, i.e., nonlinear interactions between two toroidal eigenmodes. Parallel mode wavevector also increases through coupling to the $(0, 1)$ mode, which is a weaker nonlinear interaction due to Landau damping of the $(0, 1)$ mode. Finally, the generation of the zonal flow is the weakest nonlinear interaction because the amplitude of the sidebands with $\theta_0 \neq 0$ is much smaller than the pump eigenmodes. The radial extension of the nonlinearly generated radial streamers is essentially determined by linear toroidal couplings, since the amplitude of the $(0, 1)$ mode after saturation is small compared with toroidal equilibrium variations in the poloidal direction.

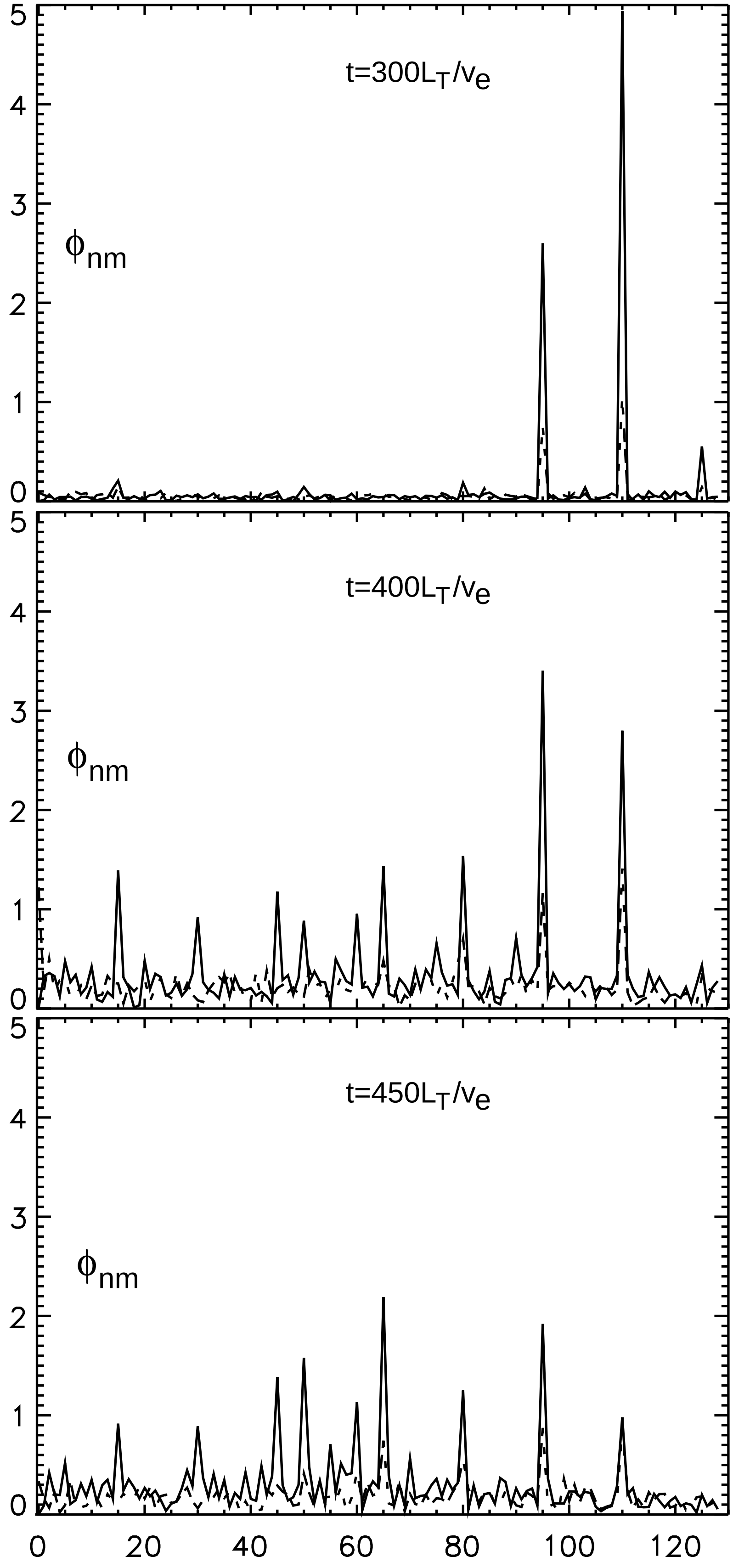

Figure 2: Toroidal mode number n spectra before and after saturation of the pump modes at $r = 0.5a$. Solid line represents the harmonics of $m = qn$; $m = qn + 1$ for dashed line.

3 Discussion and Conclusion

We note that there are accumulating evidences from first principles turbulence simulations that contradict the heuristic mixing length rule, which underlies most of the existing transport models. We have reported earlier [12] a gradual transition from Bohm to gyroBohm scaling for the ion transport driven by the ITG turbulence although the ITG eddies are isotropic. In this paper, we further demonstrate that the scaling of electron transport driven by the ETG turbulence is gyroBohm even though the size of ETG streamers scales with the device size. Given the characteristics of respective turbulence eddies, the mixing length rule would predict that the ITG transport scaling is gyroBohm and that the ETG transport scaling is Bohm. The key to reconciling the obvious contradiction is that transport is diffusive and driven by the local fluctuation intensity, rather than the eddy size. The deviation from the gyroBohm scaling in ITG transport comes from the fact that the fluctuation intensity is driven by nonlocal effects such as the turbulence spreading [12, 17, 18]. Meanwhile, the ETG fluctuation intensity is determined by the nonlinear toroidal coupling, which does not depend on the streamer (or system) size. Therefore, it is important to delineate mechanisms that determine the fluctuation intensity and the transport.

The crucial role of low-n quasi-modes as mediators in nonlinear toroidal couplings is a possible explanation of the big difference in saturation levels and transport between flux-tube and global simulations. In fact, the mode number of low-n quasi-mode is $\sim n^{1/2}$. Thus, proper radial resolution to describe their dependence imposes that the radial box size scales as $\sim n^{1/2}\rho_e$. If quasi-mode dynamics is suppressed, then only parallel mode structure modification via the $(0,1)$ mode and zonal flow dynamics can set the (much higher) saturation level of ETG turbulence. Meanwhile, the transport could be further enhanced due to the uniform safety factor q used in flux-tube simulations. Finally, since all unstable eigenmodes participate in nonlinear toroidal couplings, using a small number of toroidal eigenmodes in nonlinear simulations [8, 14] may not accurately predict the saturation amplitude.

In conclusion, global gyrokinetic particle simulations find that ETG instability saturates via nonlinear toroidal couplings, which transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths. The electrostatic ETG turbulence is dominated by nonlinearly generated radial streamers. The length of streamers scales with the device size and is much longer than the distance between mode rational surfaces or electron radial excursions. Both fluctuation intensity and transport level are independent of the streamer size.

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