LETTERS

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A fluid–kinetic hybrid electron model for electromagnetic simulations

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A fluid–kinetic hybrid electron model for electromagnetic simulations of finite-β plasmas is developed based on an expansion of the electron response using the electron–ion mass ratio as a small parameter. (Here β is the ratio of plasma pressure to magnetic pressure.) The model accurately recovers low frequency plasma dielectric responses and faithfully preserves nonlinear kinetic effects (e.g., phase space trapping). Maximum numerical efficiency is achieved by overcoming the electron Courant condition and suppressing high frequency modes. This method is most useful for nonlinear kinetic (particle-in-cell or Vlasov) simulations of electromagnetic microturbulence and Alfvénic instabilities in magnetized plasmas. [DOI: 10.1063/1.1356438]

Magnetic fluctuations in magnetized plasmas have been shown in linear theories to be key ingredients in microinstabilities,[1] drift-Alfvén wave instabilities,[2] and magnetohydrodynamic instabilities such as toroidal Alfvén eigenmodes,[3] energetic particle modes,[4] as well as the generation of magnetic pulsations in magnetosheres.[5] Nonlinear kinetic study of these electromagnetic fluctuations, meanwhile, is hindered by the difficulty of treating the dynamics of electrons whose characteristic frequency is much faster than that of the low frequency modes of interest. Specifically, the existence of high frequency modes and the electron Courant condition[6] place stringent, unnecessary numerical constraints in nonlinear kinetic simulations. As a result, most turbulence simulations have been focused on the electrostatic limit.[7,8] Recently, major efforts have been directed at developing working electron models, e.g., massless electron model (no dissipation)[9] and gyrofluid model (linear closure).[10] A fully kinetic electron model for gyrokinetic particle simulations has been proposed which extracts out the adiabatic response and solves dynamically only for the nonadiabatic response.[11] Thus, it removes the numerical noise associated with the adiabatic response, which is larger than the nonadiabatic response by the square root of electron–ion mass ratio. However, the difficulties with high frequency modes and the electron Courant condition remain to be resolved.

In this work, we develop a fluid–kinetic hybrid electron model for nonlinear kinetic simulations of low frequency turbulence in magnetized plasmas. Both the electron response and the perturbed parallel electric field are expanded based on a small electron–ion mass ratio. In the lowest order, the electrons are adiabatic and can be described by fluid equations. Thus, the high frequency modes are removed and no electron Courant condition needs to be observed. In the higher orders, the nonadiabatic response is treated kinetically with all nonlinear kinetic effects preserved. This model combines the good numerical properties of the fluid approach and the accurate kinetic effects of the fully kinetic model. It is most useful for nonlinear kinetic (particle-in-cell or Vlasov) simulations of low frequency modes, e.g., shear Alfvén waves and ion acoustic waves, in magnetized plasmas.

We consider a shearless slab with uniform magnetic field $\mathbf{B_0}=B_0\hat{\mathbf{b}}_\mathbf{0}$ and equilibrium uniform Maxwellian ions and electrons

$$f_0=n_0(2\pi)^{-1/2}v_\alpha^{-1}e^{-v_\parallel^2/2v_\alpha^2}$$

with $\alpha=i,e$ for ion and electron, respectively, $q_e=-e$, $q_i=e$, and $v_\alpha^2=T_\alpha/m_\alpha$. Assuming the usual gyrokinetic ordering, the gyrokinetic equation[12] for the perturbed distribution function $\delta f_\alpha=f_\alpha-f_0$ is

$$\frac{D}{Dt}\delta f_\alpha=-\frac{D}{Dt}f_0, \tag{1}$$

where

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$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} - \frac{\nabla \phi \times \mathbf{B}}{B^2} \right) \cdot \nabla - \frac{q_\alpha}{m_\alpha} \nabla \psi \cdot \hat{\mathbf{b}} \frac{\partial}{\partial v_\parallel},$$

$\hat{\mathbf{b}} = (\mathbf{B_0} + \delta \mathbf{B})/B$, and $\delta \mathbf{B} = \nabla \times \mathbf{A}_\parallel$. $k_\parallel \ll k_\perp$ is assumed where the wave vector is $\mathbf{k} = k_\parallel \hat{\mathbf{b}}_0 + k_\perp \hat{\mathbf{k}}_\perp$ with $\hat{\mathbf{k}}_\perp \cdot \hat{\mathbf{b}}_0 = 0$. Finite Larmor radius effects in Eq. (1) are omitted for simplicity. The parallel electric field is $E_\parallel = -\nabla \psi$, where

$$\nabla \psi \cdot \hat{\mathbf{b}} = \nabla \phi \cdot \hat{\mathbf{b}} + \frac{\partial A_\parallel}{\partial t}. \tag{2}$$

The gyrokinetic Poisson equation in the long wavelength approximation for the electrostatic potential is

$$\left(\frac{\rho_s}{\lambda_D} \right)^2 \nabla_\perp^2 \phi = -\frac{q_i \delta n_i + q_e \delta n_e}{\epsilon_0}, \tag{3}$$

where $\lambda_D^2 = \epsilon_0 T_e / n_0 e^2$, $\rho_s = c_s / \Omega_i$, $c_s = \sqrt{T_e/m_i}$, $\Omega_i = eB_0/m_i$, and $\delta n_\alpha = \int \delta f_\alpha \, dv_\parallel$. The Ampere's law for the vector potential is

$$\nabla_\perp^2 A_\parallel = -n_0 (q_i \delta u_i + q_e \delta u_e) \tag{4}$$

with $n_0 \delta u_\alpha = \int v_\parallel \delta f_\alpha \, dv_\parallel$. Equations (1)–(4) form a complete system governing the electromagnetic fluctuations in our model plasma.

Linearizing Eqs. (1)–(4) and making the ansatz $e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$, we obtain the linear dispersion relation

$$\left(\frac{\omega^2}{k_\parallel^2 v_A^2} - 1 \right) [1 + \zeta_e Z(\zeta_e) + \tau + \tau \zeta_i Z(\zeta_i)] = k_\perp^2 \rho_s^2, \tag{5}$$

where the Z function is

$$Z(\zeta_\alpha) = \frac{1}{\sqrt{\pi}} \int \frac{e^{-t^2}}{t - \zeta_\alpha} dt,$$

with $\tau = T_e / T_i$, $\zeta_\alpha = \omega / \sqrt{2} k_\parallel v_\alpha$, and Alfvén speed $v_A = B_0 / \sqrt{\mu_0 n_0 m_i}$. There are two branches of normal modes in Eq. (5): a kinetic Alfvén wave[13] with $\omega = k_\parallel v_A \sqrt{1 + k_\perp^2 \rho_s^2}$ for cold ion and adiabatic electron, an ion acoustic wave with $\omega = k_\parallel c_s$ for $\tau \gg 1$. The Alfvén wave becomes a high frequency mode (gyrokinetic version of the plasma oscillation)[6] with

$$\omega_H = \frac{k_\parallel}{k_\perp} \sqrt{\frac{m_i}{m_e}} \Omega_i$$

for $\beta_e \ll 1$, where the electron beta is

$$\beta_e = \frac{n_0 T_e}{B_0^2 / 2\mu_0}.$$

The restriction on the time step Δt for kinetic simulations of Eqs. (1)–(4) is imposed by the ω_H mode

$$\omega_H \Delta t < 1, \tag{6}$$

and the electron Courant condition

$$k_\parallel v_e \Delta t < 1. \tag{7}$$

These time step restrictions are physically unnecessary since the modes of interest (shear Alfvén waves and ion acoustic waves) typically have lower frequency $\omega \ll \omega_H, k_\parallel v_e$. Recognizing that most of the electrons behave adiabatically for these low frequency modes, a "split-weight" scheme[11] has been developed for gyrokinetic particle simulations. Instead of solving for the full perturbed distribution function, Eq. (1), this method solves dynamically the kinetic equation governing the nonadiabatic part of the electron response. This is much like the usual analytic formulation[14] and Eqs. (1)–(4) are solved exactly. Since the perturbed momentum, pressure, and even higher order moments need to be calculated from the electron distribution function which is evolved kinetically, this method has to treat very accurately the dynamics of thermal electrons that contribute predominantly to these moments. Therefore, while it removes the numerical noise associated with the adiabatic response, this scheme still needs to observe[15–17] the time step restrictions of Eqs. (6) and (7), and is subject to the numerical noise of the undamped ω_H mode. Nonetheless, the concept of treating only the nonadiabatic response dynamically does inspire the development of the present hybrid model.

We now formulate a fluid–kinetic hybrid model that can overcome the electron Courant condition and remove the unphysical ω_H mode by solving approximately, rather than exactly, Eqs. (1)–(4). Noting that the phase velocity of the shear Alfvén wave and the ion acoustic wave are typically much smaller than the electron thermal velocity, we can expand the linear solution of Eq. (1) for the electron

$$\delta f_e = \frac{k_\parallel v_\parallel}{k_\parallel v_\parallel - \omega} \frac{e\psi}{T_e} f_0 = \frac{e\psi}{T_e} f_0 \left(1 + \frac{\omega}{k_\parallel v_\parallel} + \cdots \right).$$

In this expansion, the small parameter $\delta_m \ll 1$ is the ratio of the wave phase velocity to the electron thermal velocity, i.e., for the shear Alfvén wave

$$\frac{\omega}{k_\parallel v_\parallel} \sim \frac{v_A}{\sqrt{2} v_e} = \left(\frac{m_e}{\beta_e m_i} \right)^{1/2} \equiv \delta_m,$$

and for the ion acoustic wave, $\delta_m \equiv \sqrt{m_e/m_i}$. δ_m represents the deviation from the adiabatic response. When $\delta_m < 1$, we can expand Eqs. (1)–(4) based on δ_m and solve them order by order. Formally, the parallel electric field potential ψ and distribution function f_e are expanded,

$$\psi = \psi^{(0)} + \psi^{(1)} + \cdots, \quad f_e = f_0 e^{e\psi/T_e} + \delta g_e^{(1)} + \cdots.$$

In the lowest order, $\psi = \psi^{(0)}$ and $f_e = f_0 e^{e\psi^{(0)}/T_e}$, and all electrons are adiabatic and can be described by a fluid model. In particular, the density is governed by the continuity equation which can be obtained from Eq. (1),

$$\frac{\partial \delta n_e}{\partial t} = -\mathbf{B} \cdot \nabla \frac{n_0 \delta u_e}{B}. \tag{8}$$

The vector potential evolves according to Eq. (2),

$$\frac{\partial A_\parallel}{\partial t} = \hat{\mathbf{b}} \cdot \nabla (\psi - \phi). \tag{9}$$

These two dynamical equations are closed by field equations, i.e., the Poisson equation,

$$\left(\frac{\rho_s}{\lambda_D} \right)^2 \nabla_\perp^2 \phi = -\frac{q_i \delta n_i + q_e \delta n_e}{\epsilon_0}, \tag{10}$$

and Ampere's law

$$n_0 q_e \delta u_e = -\nabla_\perp^2 A_\parallel - n_0 q_i \delta u_i . \tag{11}$$

The lowest order solution for the parallel electric field is

$$e^{e\psi^{(0)}/T_e} - 1 = \frac{\delta n_e}{n_0}. \tag{12}$$

Equations (8)–(12) are a complete set for the fluid electron model in the lowest order in δ_m.

In the first order in δ_m, we treat the nonadiabatic response using the electron kinetic equation, Eq. (1), with the perturbed field from the lowest order solution

$$\frac{D}{Dt}\delta g_e^{(1)} = f_0 \frac{e}{T_e} e^{e\psi^{(0)}/T_e}\left(-\psi_t^{(0)} + \frac{\nabla\phi\times\mathbf{B}}{B^2}\cdot\nabla\psi \right). \tag{13}$$

With $\psi_t^{(0)} = \partial\psi^{(0)}/\partial t$ which can be evaluated from the lowest order solution of Eqs. (8) and (12),

$$\frac{e}{T_e}\psi_t^{(0)} = -\mathbf{B}\cdot\nabla\frac{\delta u_e}{B}. \tag{14}$$

The parallel electric field with the first order correction $\psi = \psi^{(0)} + \psi^{(1)}$ can be obtained from the f_e expansion

$$e^{e\psi/T_e} - 1 = \frac{\delta n_e}{n_0} - \frac{\delta n_e^{(1)}}{n_0}, \tag{15}$$

with $\delta n_e^{(1)} = \int \delta g_e^{(1)} dv_\parallel$. Equations (13)–(15) form a complete system for the first order correction of the nonadiabatic response. This procedure can be repeated to achieve accuracy to higher order in δ_m.

We now examine the linear dielectric properties of the hybrid electron model with first order accuracy in δ_m, Eqs. (8)–(15). Ion dynamics are treated using the usual gyrokinetic equation, Eq. (1). Linearizing these equations, we obtain the dispersion relation

$$\left(\frac{\omega^2}{k_\parallel^2 v_A^2} - 1\right)\left[\frac{1}{1-\zeta_e Z(\zeta_e)} + \tau + \tau\zeta_i Z(\zeta_i)\right] = k_\perp^2\rho_s^2. \tag{16}$$

As expected, this dispersion relation of the hybrid model is a result of a small parameter expansion of the drift kinetic dispersion relation, Eq. (5),

$$\frac{1}{1+\zeta_e Z(\zeta_e)} = 1 - \zeta_e Z(\zeta_e) + \cdots, \quad \text{for } \zeta_e \sim \delta_m \ll 1.$$

Note that the crucial kinetic effect, Landau damping, is retained rigorously in this expansion. This hybrid model can be regarded as a nonlinear closure scheme. Unlike the gyrofluid closure,[10] which only treats linear wave–particle interactions, i.e., phase mixing, the hybrid model preserves faithfully nonlinear kinetic effects, e.g., particle trapping by waves. In the limit of small δ_m, the hybrid model recovers the exact kinetic results. What is more, it has superior numerical properties as compared to the fully kinetic model. First, there are only two branches of normal modes in Eq. (16): the shear Alfvén wave and the ion acoustic wave. The unphysical high frequency ω_H mode is explicitly removed from the hybrid model because of the adiabatic electron response in the lowest order of expansion. Thus, the numerical noise and the time step restriction [Eq. (6)] associated with this mode are removed analytically. This results in a lesser required number of particles and a larger time step in the simulations. Second, the hybrid model overcomes the electron Courant condition of Eq. (7). In the lowest order with the fluid equations, the time step restriction is the shear Alfvén wave or the ion acoustic wave frequency ω_A,

$$\omega_A \Delta t < 1. \tag{17}$$

In the higher order with kinetic corrections, the time step restriction is the transit time of resonant electrons with a low velocity $v_\parallel \sim v_A$, c_s. This is identical to the time step of Eq. (17). Therefore, unlike the fully kinetic approach[15–17] where numerical accuracy requires the observation of Courant condition, our hybrid model preserves accurate electron response without the constraint of the Courant condition for both kinetic Alfvén wave and ion acoustic wave.

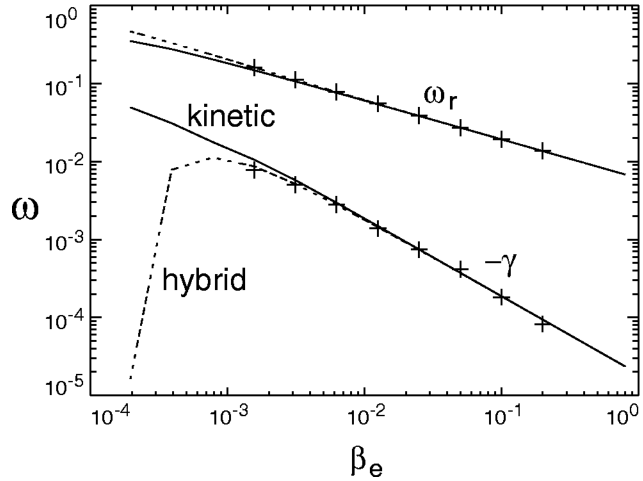

FIG. 1. Kinetic Alfvén wave real frequencies ω_r/Ω_i (upper curves) and damping rates $-\gamma/\Omega_i$ (lower curves) vs electron beta. Solid lines are kinetic results, dashed lines are hybrid model results, and "+" points are simulation results.

While the hybrid electron model is valid for any low frequency modes including kinetic Alfvén waves and ion acoustic waves, here we study the kinetic shear Alfvén wave in a shearless slab to demonstrate the utility of the hybrid model. In these gyrokinetic particle simulations, the electron dynamics is treated using Eqs. (8)–(15) and the ion contribution is omitted for simplicity (the ion acoustic wave is suppressed). The corresponding analytic dispersion relations of Eqs. (5) and (16) are shown in Fig. 1. The hybrid model results (dashed line) agree with the exact kinetic results (solid lines) for $\beta_e > m_e/m_i$, i.e., $\delta_m < 1$. The simulation results ("+") also agree very well with the analytic results of the hybrid model. The simulation parameters are $m_i/m_e = 1837$, $k_\perp\rho_s = 0.4$, $k_\parallel/k_\perp = 0.01$, and $0.001 < \beta_e < 0.2$. The number of spatial grids is $N_g = 64$, the number of electrons is $N_p = 10\,000$, and time step is $\omega_A\Delta t = 0.1$ (which is required for integration of a simple oscillator with frequency ω_A using a second order Runge–Kutta method). For the high-β_e cases, this time step violates the electron Courant condition of Eq. (7) and the grid size is also larger than the collisionless elec-

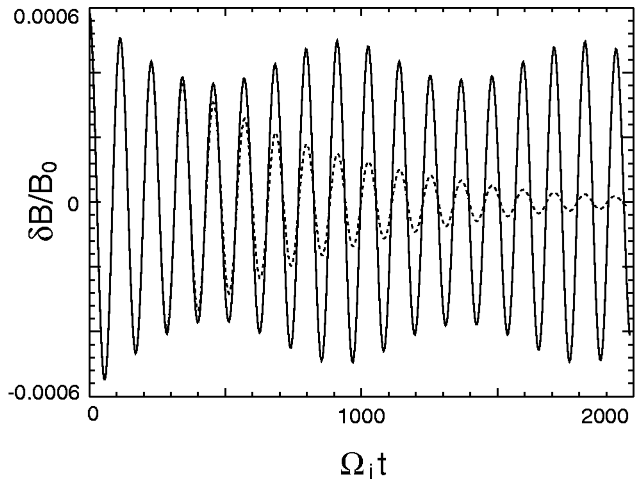

FIG. 2. Nonlinear simulation of kinetic Alfvén wave exhibits oscillation in amplitude of perturbed magnetic field (solid line), while linear simulation shows exponential decay (dashed line).

tron skin depth, yet accurate results are obtained. Thus the issue of resolving the skin depth in a fully kinetic model[16,17] is removed from our hybrid model.

The nonlinear simulation result for $\beta_e = 0.0125$ is shown in Fig. 2. The amplitude of the magnetic perturbation is oscillatory (solid line) due to the trapping of resonant electrons. The bounce frequency ω_b in Fig. 2 is close to the theoretical estimate of $\omega_b = k_\parallel v_e \sqrt{e\psi/T_e}$ (in the simulation $e\psi/T_e = 0.0025$). At the minimum of the field perturbation, the electron distribution function also exhibits population inversion around the resonant point. In an otherwise identical linear simulation (dashed line) the magnetic perturbation decays exponentially. The observation of particle trapping in the nonlinear simulation shows that the hybrid model faithfully preserves this important nonlinear kinetic effect.

It is clear from Fig. 1 that the electromagnetic version of the hybrid electron model is not valid for $\beta_e \leqslant m_e/m_i$. In this regime, the shear Alfvén wave has a phase velocity faster than the electron thermal velocity. Since simulations need to resolve the fast time scale of this inertial Alfvén wave anyway, electrons do not place any additional numerical constraint. Therefore, a conventional fully kinetic treatment is warranted. In such low beta plasmas, we can also extend the hybrid model to the electrostatic limit which is of more practical interest. Here, the magnetic perturbation is negligible in most cases because the fluctuating current is very small and electrostatic fluctuations dominate the dynamics. Thus for $\beta_e \leqslant m_e/m_i$, simulations can be reduced to an electrostatic one (effectively $\beta_e \sim 0$). Here the smallness parameter is the ratio of sound speed to electron thermal speed, $\delta_m \equiv \sqrt{m_e/m_i}$. In the lowest order in δ_m, electrons are adiabatic and we only need Eq. (10). The next order nonadiabatic correction is governed by Eq. (13). As expected, the linear dispersion relation of this electrostatic system has only one normal mode: the ion acoustic wave. The unphysical ω_H mode, which has a thermal fluctuation level much higher than that of the ion acoustic wave,[6] is again explicitly removed. Furthermore, the electron Courant condition can be violated. These are the advantages of the hybrid model in the electrostatic limit as compared to the electrostatic split-weight scheme[11] which is fully kinetic.

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