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Turbulence spreading into the linearly stable zone and transport scaling

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Abstract

We study the simplest problem of turbulence spreading corresponding to the spatio-temporal propagation of a patch of turbulence from a region where it is locally excited to a region of weaker excitation or even local damping. A single model equation for the local turbulence intensity, $I(x, t)$, includes the effects of local linear growth and damping, spatially local nonlinear coupling to dissipation and spatial scattering of turbulence energy induced by nonlinear coupling. In the absence of dissipation, front propagation into the linearly stable zone occurs with the property of rapid progression at small t, followed by slower sub-diffusive progression at late times. The turbulence radial spreading into the linearly stable zone reduces the turbulent intensity in the linearly unstable zone and introduces an additional dependence on the $\rho^* \equiv \rho_i/a$ to the turbulent intensity and the transport scaling. These are in broad, semi-quantitative agreement with a number of global gyrokinetic simulation results with zonal flows and without zonal flows. Front propagation stops when the radial flux of fluctuation energy from the linearly unstable region is balanced by local dissipation in the linearly stable region.

1. Introduction

Achieving an understanding of turbulent transport is necessary for the design of an economical advanced tokamak fusion reactor. In recent years, progress in experiment, theory and computation has been dramatic, and yet the Holy Grail of predictive capacity by other than brute force, case-by-case direct numerical simulation, remains elusive. Serious challenges remain due to the fact that virtually all models of fluctuation levels and turbulent transport are built on an assumption of local balance of linear growth with linear damping and nonlinear coupling to dissipation. Here, 'local balance' refers to balance at a point or in a region comparable in extent with the modal width. Such models thus necessarily exclude mesoscale dynamics, which refers to dynamics on scales larger than a mode or integral scale eddy size but smaller than the system size or profile scale length. In particular, transport barriers, avalanches, heat and particle pulses all are mesoscale phenomena [1–3].

In this paper, we identify and study in depth the simplest, most minimal problem in the mesoscale dynamics category before proceeding to consider more complicated examples. In this case, the 'minimal problem' is that of the spatio-temporal propagation of a patch of turbulence from a region where it is locally excited to a region of weaker excitation or even local damping. This process can be described by a single model equation for the local turbulence intensity, $I(x,t)$, which includes the effects of local linear growth and damping, spatially local nonlinear coupling to dissipation and spatial scattering of turbulence energy induced by nonlinear coupling. These effects combine to give an energy equation loosely of the form

$$\frac{\partial I}{\partial t} - \frac{\partial}{\partial x}\chi(I)\frac{\partial I}{\partial x} = \gamma(x)I - \alpha I^{1+\beta}, \tag{1}$$

the terms of which correspond to nonlinear spatial scattering (i.e. typically $\chi(I) \sim \chi_0 I^{\beta}$, where $\beta = 1$ for weak turbulence and $\beta = \frac{1}{2}$ for strong turbulence (see, for instance [4] and references therein)), linear growth and damping, and local nonlinear decay, respectively. Here, α is a nonlinear coupling coefficient. Note that α and χ_0 could be functions of radius. This energy equation is the irreducible minimum of the model, to which additional equations for other fields, and contributions to dynamics that feedback on I, may be added. Note that the above energy equation manifests the crucial effect of spatial coupling in the nonlinear diffusion term. This implies that the integrated fluctuation intensity in a region of extent $\triangle$ about a point x (i.e. $\int_{x-\triangle}^{x+\triangle} I(x')\,\mathrm{d}x'$) can grow, even for negative $\gamma(x)$, so long as $\chi(I)\partial I/\partial x|_{x-\triangle}^{x+\triangle}$ is sufficiently large. Alternatively, I can decrease, even for positive $\gamma(x)$, should $\chi(I)\partial I/\partial x|_{x-\triangle}^{x+\triangle}$ be sufficiently negative. Thus, the profile of fluctuation intensity is crucial to its spatio-temporal evolution. These simple observations nicely illustrate the failure of the conventional local saturation paradigm [5] and strongly support the argument that propagation of turbulence energy is a crucial, fundamental problem in understanding confinement scalings for fusion devices in which growth and damping rate profiles vary rapidly in space.

While the radial spreading of turbulence has been widely observed in previous global nonlinear gyrokinetic and mode coupling simulations [6–8], its significance has not been widely recognized. Its effect on turbulent transport scaling has been addressed only recently [9]. In our work [9], we conjectured that the turbulence radial spreading into the linearly stable zone can reduce the turbulent intensity in the linearly unstable zone and introduce an additional dependence on the $\rho^* \equiv \rho_i/a$ to the turbulent intensity, which is otherwise determined by local physics. Since the ion thermal diffusivity, χ_i, was observed to be proportional to I for the weak turbulence case in the previous gyrokinetic simulation [10], this in turn can cause a deviation from gyroBohm transport scaling, which was expected from the local turbulence characteristics (i.e. a radial correlation length about $\Delta r \simeq 7\rho_i$, independent of the system size). The basic features of an analytic, dynamical model of turbulence [11] has been previously published in [12]. In this paper, we present a more complete version and more detailed comparisons with gyrokinetic simulation results [9].

The remainder of this paper is organized as follows. In section 2, we propose a nonlinear diffusion equation as the simplest model for the problem of turbulence spreading and present detailed analytic solutions of this nonlinear diffusion equation. Section 3 contains the effects of radial spreading on transport scaling and comparisons with recent global gyrokinetic simulation results. Section 4 consists of conclusions.

2. Dynamics of turbulence spreading

Another aspect of the dynamics that falls outside the traditional 'local balance' paradigm of Kadomtsev [5] is illustrated by the equation for $I(x)$. First, turbulence energy propagation is intrinsically nondiffusive since $\chi(I)$ increases with I. This is easily seen by observing that for turbulent diffusion $\chi(I) = \chi_0 I^\beta$, so that the natural diffusive scalings for the width of a turbulent patch are $l^2 \sim \chi_0 I^\beta t$ and $Il = Q_0 \equiv I_0 l_0$ in the absence of growth or dissipation. It thus follows that the self-similarity variable is $x/l(t) = x/(\chi_0 Q_0^\beta t)^{1/(2+\beta)}$, and so a turbulent patch spreads as $\triangle x \sim (\chi_0 {Q_0}^\beta t)^{1/(2+\beta)}$. Contrary to conventional wisdom, this actually corresponds to sub-diffusive propagation, which has the property of rapid progression at small t, followed by slower progression at late times. Thus, the rapid re-adjustment and spatial spreading of turbulence intensity profiles observed in several gyrokinetic particle simulations [6, 7, 9] are quite likely symptoms of turbulence propagation.

Focusing on the role of nonlinear diffusion in the weak turbulence regime (with $\beta = 1$ in equation (1)) as observed in the gyrokinetic simulations [9, 10], we begin our analysis using the following nonlinear partial differential equation:

$$\frac{\partial}{\partial t} I = \gamma(x) I - \alpha I^2 + \chi_0 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I \right). \tag{2}$$

Here, $I \equiv \langle (e\phi/T_e)^2 \rangle$ is the envelope of the turbulence intensity in the energy containing range in k_y excluding the zonal flows with $k_y = k_z = 0$, $\gamma(x)$ is the 'local' excitation rate of the instability, for instance the ion temperature gradient (ITG) driven mode, x is a radial coordinate, α is the local nonlinear coupling term [4]. For a diffusion coefficient of turbulence χ_i, we take $\chi_i \equiv \chi_0 I$ with an explicit proportionality to I as observed in the previous global nonlinear gyrokinetic simulation [10]. We note that equation (2) is similar to a generalized K–ϵ model for its direct inverse process, namely transport barrier propagation and broadening [13–15].

First, we review the local solution. In the absence of the nonlinear radial diffusion given by the last term, we can integrate equation (2) in time with an initial profile $I(x, 0) < \gamma(x)/\alpha$ to obtain

$$I(x,t) = \left(\frac{\gamma(x)}{\alpha} \right) \left(1 + \frac{(\gamma(x)/\alpha) - I(x,0)}{I(x,0)} \, \mathrm{e}^{-\gamma(x)t} \right)^{-1}. \tag{3}$$

Equation (3) describes the time evolution of the intensity towards a nonlinear saturation. In the region $\gamma > 0$, I initially grows exponentially with a linear growth rate $\gamma(x)$ and then saturates at a finite level given by $\gamma(x)/\alpha$. However, in the region where $\gamma < 0$, this local solution predicts that the fluctuation vanishes ($I \to 0$).

Now, we study in detail how the nonlinear diffusion term in equation (2) allows fluctuations to spread into a zone where $\gamma < 0$. In the region where $\gamma(x) \simeq 0$ and $I \ll 1$, the first two terms on the rhs can be ignored. Equation (2) simplifies to the following nonlinear partial differential equation, which is also known as the modified porous-medium equation [16].

$$\frac{\partial}{\partial t} I_0 = \chi_0 \frac{\partial}{\partial x}\left(I_0 \frac{\partial}{\partial x} I_0 \right). \tag{4}$$

We consider a smooth radially varying linear excitation rate profile $\gamma(x)$ that is similar in shape to the ones used in a global gyrokinetic simulation of ITG instabilities discussed in [6, 9] in shape. As shown in figure 1, $\gamma > 0$ in the middle for $|x - x_i| < W$, and x_i is the position where $\gamma(x)$ is maximum.

Then γ decreases monotonically towards the axis and the edge, becomes 0 at $x = x_i - W$ and at $x = x_i + W$ and becomes negative for $|x - x_i| > W$. In this paper, we only consider the case where the background pressure and $\gamma(x)$ do not change in time. This makes comparisons

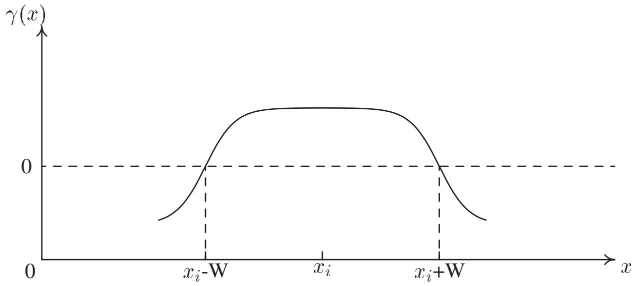

Figure 1. Local excitation rate, $\gamma(x)$, as a function of radius.

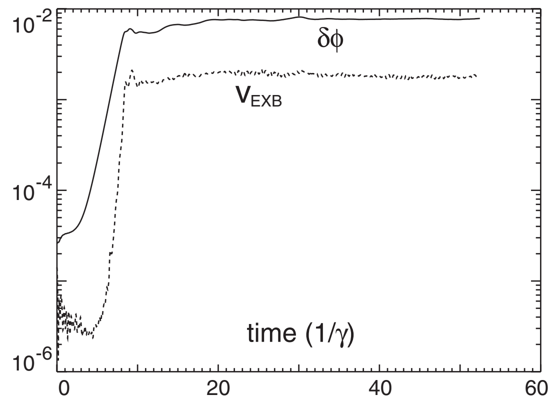

Figure 2. The volume-integrated fluctuation intensity ($\delta\phi$) saturates in magnitude as the self-generated zonal flow ($V_{\boldsymbol{E}\times\boldsymbol{B}}$) grows to a saturated amplitude. The radial spreading of turbulence occurs from approximately $t = 8/\gamma$, while the volume-integrated fluctuation intensity stays approximately a constant in time.

with the recent gyrokinetic simulations [12] more straightforward. For more challenging problems such as the formation of transport barriers, where the disparity in timescales becomes less obvious, one needs to extend the theory to a multifield nonlinear system [14, 15, 17, 18] in which evolutions of $\boldsymbol{E} \times \boldsymbol{B}$ flows and the pressure gradient are included [19]. Gyrokinetic simulation in [12] shows that the volume integrated fluctuation intensity saturates in magnitude as the self-generated zonal flow grows to a saturated amplitude as discussed in detail in [20,21]. Then, the radial spreading of turbulence occurs from approximately $t = 8/\gamma$, while the volume integrated fluctuation intensity stays approximately a constant in time as shown in figure 2. We focus our studies on this later phase. For an initial profile

$$I_0(x,0) = \frac{\epsilon}{W}\left(1 - \frac{(x-x_i)^2}{W^2}\right) H(W - |x - x_i|),$$

equation (4) has an exact solution [16]:

$$I_0(x,t) = \frac{\epsilon}{(6\epsilon\chi_0 t + W^3)^{1/3}}\left(1 - \frac{(x-x_i)^2}{(6\epsilon\chi_0 t + W^3)^{2/3}}\right) H\big((6\epsilon\chi_0 t + W^3)^{1/3} - |x - x_i|\big), \qquad (5)$$

where ϵ is the volume-integrated intensity, H is a Heaviside function. Equation (5) shows that in the absence of linear or nonlinear damping (the first term and the second term on the rhs of equation (2)), a fluctuation front at $x = x_i + (W^3 + 6\epsilon\chi_0 t)^{1/3}$ will propagate beyond

$x_0 \equiv x_i + W$ indefinitely. (The same comment applies to another front at $x = x_i - W$, which propagates to the left.) The time evolution of I_0, exhibiting a localized front-like solution described by equation (5), is plotted in figure 2.

We note that the short time behaviour of propagation can be characterized by $(x-x_0)_{\rm front} \simeq U_x t$ after expanding in $(x - x_0)_{\rm front}/W$, with $U_x = 2\epsilon\chi_0/W^2$. This apparent ballistic behaviour in the short term is mainly a consequence of the fact that $\Delta \ll W$, and it is not difficult to derive from other theoretical considerations (figure 2). It is obvious that the expression for U_x from our nonlinear theory is qualitatively different from the radial group velocity of a drift wave. This is one of the signatures that distinguish our nonlinear diffusion theory from the other models, which heavily rely on the specific properties of the drift wave linear dispersion relation [8] or on nonlinear enhancement of dispersion [22] of the four mode system consisting of toroidal eigenmodes and zonal flows [23]. The long term behaviour of propagation is sub-diffusive, as discussed previously,

$$\triangle x \sim (\chi_0 Q_0 t)^{1/3}. \tag{6}$$

The relevant question here is, What is the physics mechanism responsible for a saturation of the fluctuation spreading in a linearly stable zone? At a conceptual level, most existing models of turbulent transport are based on a local balance of excitation and dissipation. For instance, the commonly used local saturation condition *á la* Kadomtsev [5] in which one balances the linear growth rate, $\gamma > 0$, and the local nonlinear damping (i.e. $\gamma \sim k_\perp^2 D_{\rm turb}$) does not apply here. We expect that the fluctuation front would cease to propagate if the fluctuation energy flux due to radial propagation into the linearly stable zone is balanced by dissipation (figure 4). First, we consider the case where the linear damping near the propagating front ($\gamma(x) \simeq -|\gamma'|(x-x_0)$) is strong enough to play a dominant role in limiting the radial spreading. The scaling for Δ can be obtained by balancing the time required for linearly damping the fluctuation at $x = x_0 + \Delta$, i.e. $T_{\rm damp} \sim 1/(|\gamma'|\Delta)$, against the time required for the front to propagate a distance Δ (which is shorter than the system size), i.e. $T_{\rm prop} \simeq \Delta/U_x$. The resulting scaling with respect to the

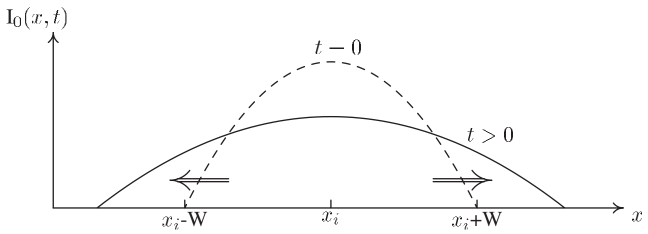

Figure 3. Equation (5) describes a propagating localized front-like solution.

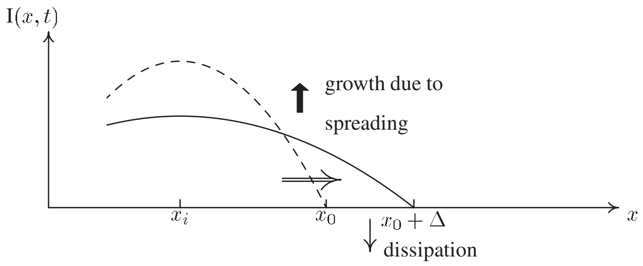

Figure 4. Fluctuation front ceases to propagate if the fluctuation energy flux due to radial propagation is balanced by dissipation.

damping rate is weaker ($\Delta \propto |\gamma'|^{-1/2}$) than that based on a heuristic argument based on the linear toroidal coupling, $\Delta \propto \gamma_{\mathrm{damp}}^{-1}$ [24]. Quantitatively, the front stops propagating when the width of spreading, Δ, satisfies the following condition,

$$\frac{\partial}{\partial T}\int_{x_0}^{x_0+\Delta} \mathrm{d}x\, I_0(x, T) = -\int_{x_0}^{x_0+\Delta} \mathrm{d}x\, \gamma(x, T) I_0(x, T), \tag{7}$$

which yields the expression for the width of the radial spreading, Δ:

$$\Delta^2 \simeq \frac{12\epsilon\chi_0}{|\gamma'|W^2}. \tag{8}$$

As expected, higher fluctuation intensity in a linearly unstable zone (ϵ) enhances the radial spreading, while a strong linear damping reduces it.

We note that depending on the scaling of ion thermal transport in the absence of turbulence spreading, $\chi \propto \epsilon\chi_0/W$, equation (8) predicts different scalings of the radial spreading, Δ, with respect to the system size. These are in broad semi-quantitative agreements with observations from several global gyrokinetic simulation results [6, 7, 9, 25, 26]. For the cases considered in simulations where both W and the linear growth rate profile length scale with a minor radius rather than ρ_i, we obtain $\Delta \propto \rho_i$ for gyroBohm χ_i in the absence of turbulence spreading, and $\Delta \propto \sqrt{W\rho_i}$ for Bohm χ_i in the absence of turbulence spreading. We discuss details in the next section. If the linear damping is sufficiently weak, the nonlinear coupling to damped modes in the dissipative range could act to limit the range of turbulence spreading in space. The nonlinear damping is a manifestation of the nonlinear mode coupling in $\boldsymbol{k}$-space, including the interaction with zonal flows [23, 27, 28]. However, the incoherent noise, which is ignored in our nonlinear diffusion model in this paper, can also be produced from nonlinear mode coupling and will be likely to significantly enhance the range of spreading, in particular near low order rational surfaces. This effect will be studied in a future publication. With this caveat in mind, our estimation below, which relies on the coherent nonlinear mode coupling only, should be taken as an underestimate of turbulence spreading. Following a straightforward dimensional analysis, we find that $\Delta \propto \sqrt{\chi_0/\alpha}$ when the nonlinear damping saturates the fluctuation propagation.

3. Effects of turbulence spreading on transport scalings

While the radial spreading of turbulence has been widely observed in the previous global nonlinear simulations [8, 7, 6, 25, 26, 29], its effect on turbulent transport scaling has not been addressed until recently [9]. In our work [9], we conjectured that the radial turbulence spreading into the linearly stable zone can reduce the turbulent intensity in the linearly unstable zone and introduce an additional dependence on the $\rho^* \equiv \rho_i/a$ to the turbulent intensity, which is otherwise determined through a local physics. Since $\chi_i \propto I$ for the weak turbulence case as observed in the previous gyrokinetic simulations [10], this in turn can cause a deviation from the gyroBohm transport scaling, which was expected from the local turbulence characteristics (i.e. $\Delta r \simeq 7\rho_i$) only. The basic features of the analytic dynamical model of turbulence spreading [11] and progress towards a theoretical underpinning has been published in [12, 30]. In this section, we present the specific predictions from our present nonlinear diffusion model and compare them with the recent gyrokinetic simulation results [31]. Equation (5) shows that in the absence of dissipation, a fluctuation front movement into the linearly stable zone by a radial width Δ reduces the peak fluctuation intensity at $x = x_i$ to

$$I(x_i, T) = \frac{I(x_i, 0)}{1 + \Delta/W}. \tag{9}$$

Since we are interested in the regime where $\chi_i \propto I$ [10], we obtain

$$\chi_i = \frac{\chi_{i0}}{1 + \Delta/W}, \tag{10}$$

where χ_{i0} is the ion thermal diffusivity in the absence of the radial spreading of turbulence.

Based on the following estimation, we now argue that the reduction in the peak fluctuation intensity due to dissipation without the radial spreading of fluctuations is exponentially small in the limit $W/\rho_i \gg 1$. We seek a steady state solution of equation (2) for a $\gamma(x)$ profile that is piece-wise constant in radius,

$$-\chi_0 \frac{\partial}{\partial x}\left(I \frac{\partial}{\partial x} I\right) = \gamma(x) I - \alpha I^2, \tag{11}$$

where $\gamma(x) = \gamma_0 > 0$ for $|x - x_i| < W$, and $\gamma(x) = -|\gamma_d| < 0$ for $|x - x_i| > W$. One finds that for a piece-wise constant γ profile, equation (11) can be written as a perfect derivative in x by multiplying both sides by $I(\partial I/\partial x)$.

$$-\frac{\chi_0}{4}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} F\right)^2 = \frac{2}{3}\gamma \frac{\partial}{\partial x} F^{3/2} - \frac{\alpha}{2}\frac{\partial}{\partial x} F^2, \tag{12}$$

here $F = I^2$. An integration of equation (12) in radius with boundary conditions $(\partial/\partial x) I(x) = 0$ at $x = x_i$, and $I(x) = 0$ for $|x - x_i| \to \infty$ guarantees a steady state solution for a large enough system size. Here, we estimate the reduction in peak turbulence intensity (at $x = x_i$) due to a dissipation at $|x - x_i| > W$ in the absence of radial spreading. By taking a limit of very strong linear damping, $|\gamma_d| \to \infty$, we obtain the following relation by integrating equation (12) from $x = x_i$ to x,

$$\frac{\chi_0}{4}\left(\frac{\partial}{\partial x} F\right)^2 = \frac{2}{3}\gamma_0 (F(x_i)^{3/2} - F^{3/2}) - \frac{\alpha}{2}(F(x_i)^2 - F^2). \tag{13}$$

By integrating equation (13) in F from $F(x_i + W) = 0$ to $F(x_i)$ after undoing the square, we obtain the following integral relation between $W/\Delta r$ and $F(x_i)$,

$$\int_0^{F(x_i)} \frac{\mathrm{d}F}{\sqrt{(8\gamma_0/3\chi_0)(F(x_i)^{3/2} - F^{3/2}) - (2\alpha/\chi_0)(F(x_i)^2 - F^2)}} = W, \tag{14}$$

here we used the fact that $I(x) = 0$ at $|x - x_i| = W$. A simple inspection shows that the integral exhibits a logarithmic divergence for the $W \to \infty$ limit in which the fluctuation intensity saturation level is determined by the local physics, that is, $F(x_i) \simeq \gamma_0/\alpha$. An asymptotic analysis yields

$$F(x_i) \simeq \frac{\gamma_0}{\alpha}\left(1 - \mathrm{O}\left(\exp\left(-\frac{W}{\Delta r}\right)\right)\right), \tag{15}$$

where $\Delta r \equiv \sqrt{\chi_0/\alpha}$ is the radial correlation length which scales with the ion gyroradius. We, therefore, conclude that the reduction in intensity in the linearly unstable zone at the middle should originate mainly from the fluctuation spreading.

Having established a relation between the radial spreading and transport scaling in equation (10), we discuss the observations from previous global gyrokinetic simulations and possible relevance of our theoretical predictions to these simulations [6, 7, 9]. To our knowledge, the first significant numerical study addressing turbulence spreading has been performed in the context of a global mode coupling analysis of a toroidal drift wave [8]. It was observed that the linear toroidal coupling of different poloidal harmonics played a dominant role in the convective propagation of fluctuations into a region with a zero level background of fluctuations in most parameter regimes. It is worthwhile to note that [8] was published before the important role of self-generated zonal flows in regulating turbulence in a toroidal geometry was fully realized [20]. In a fashion similar to that in which mean $\boldsymbol{E} \times \boldsymbol{B}$ flow shear causes decorrelation of turbulence in the radial direction [32, 33], the random shearing by zonal flows [21, 27], which has not been included in [8], would make the linear toroidal coupling much weaker. This is shown by the measured reduction in the radial correlation length of fluctuations [21] as radially global toroidal eigenmodes get destroyed by the zonal flows in gyrokinetic simulations [20]. Garbet *et al* [8] reports that, when the linear toroidal coupling is suppressed, the fluctuation propagation due to nonlinear mode coupling has been observed to be close to a diffusive process. If we consider the limited numerical accuracy of the time of flight measurements employed in [8], this result does not seem to contradict our prediction of sub-diffusive propagation as discussed at the beginning of section 2 [24]. In the strong turbulence regime considered in [8] where $\beta = \frac{1}{2}$, we get $\triangle x \sim (\chi_0 {Q_0}^{1/2} t)^{2/5}$, which is difficult to differentiate from the diffusive behaviour of $\triangle x \propto t^{1/2}$.

We expect that in the presence of zonal flows, the nonlinear radial diffusion remains a robust mechanism responsible for the turbulence spreading, while the communication between different poloidal harmonics becomes relatively ineffective. Now we briefly describe recent nonlinear gyrokinetic particle-in-cell simulations [9, 12] where the self-generated zonal flows regulate turbulence. These nonlinear gyrokinetic particle-in-cell simulations have been performed with gyrokinetic turbulence code (GTC) [20]. Global field line following coordinates have been used to take advantage of fluctuation structures that align with equilibrium magnetic field lines. However, unlike many quasi-local codes in flux-tube geometry, GTC does not rely on the ballooning mode formalism, which becomes dubious in describing meso-scale structures [1–3] in fully developed turbulence. Therefore, realistic radial profile variations can be included straightforwardly.

The simulation parameters used are based on those from [34] which uses a simple characterization of a typical DIII-D H-mode core plasma. A system size (ρ^*) scan is then carried out with other dimensionless parameters fixed with a radial variation of ITG included as shown in [12]. The peak value of R/L_{T_i} in the middle is 6.9, which is well above marginality. Towards the axis and the edge the gradient gets weaker. While the gyrokinetic particle simulations in the mid-90s [7, 6, 25, 26] showed that radially elongated eddies, which are closely related to the global toroidal eigenmodes, partially survived in the nonlinear phase, more recent global simulations showed that self-generated zonal flows break up the radially elongated eddies [20] and isotropize the $\boldsymbol{k}$ spectrum of fluctuations. The broadening in the k_r spectrum of turbulence [21] in the presence of zonal flows in the simulation is a consequence of random shearing [21, 27, 35]. Therefore, we think it is reasonable to expect that linear toroidal coupling effects become weaker in the presence of zonal flows. More detailed two-point correlation analysis of simulation data indeed shows that the correlation length of fluctuations scales with ion gyroradius, $\Delta r \simeq 7\rho_i$, and these are invariant with respect to the system size. In the nonlinear phase of simulations, fluctuations spread radially. Their radial extent is approximately $25\rho_i$ or 3–4 radial correlation lengths in each direction. Interestingly, it is independent of the system size, as inferred from figure 5 [31]. Using the values of W, ϵ, χ_0, γ' and α from simulations, we estimate the predicted scalings and values of Δ given in equation (8). From the simulations described above, $W = x_0 - x_i = 0.75a - 0.5a = 0.25a$, $\chi_i(0.5a, T) = 1.35(cT_i/eB)(\rho_i/L_{T_i})$ and $\gamma(x_i) = 0.033(v_{T_i}/L_{T_i})$ for $k_\theta \rho_i = 0.2$, where I peaks at nonlinear saturation. Note that 0.033 is measured from the linear phase of gyrokinetic simulation and is rather a small coefficient coming from kinetic effects not captured in a heuristic dimensional argument based on a simple fluid picture. We note that radial diffusion of ion heat is not always same as that of fluctuation intensity. For the estimation of γ', we use a scaling that $\gamma \propto k_\theta \rho_i (v_{T_i}/R_0)(R_0/L_{T_i} - R_0/L_{\rm crit})$ for small $k_\theta \rho_i$ near the local

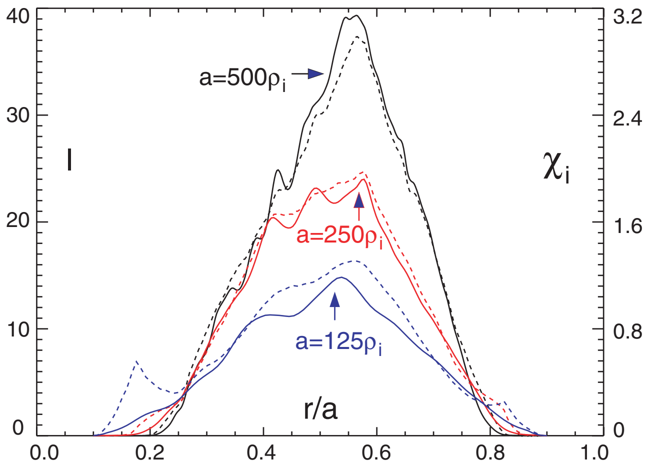

Figure 5. Fluctuation intensity profiles (I in ——) and ion thermal diffusivity (χ_i in $\cdots\cdots$), both in gyroBohm units after nonlinear saturation for $a/\rho_i = 125, 250$ and 500: as the system size gets larger, the extent of radial spreading of turbulence into the linearly stable zone gets narrower relative to the system size. It scales with the ion gyroradius $\sim 25\rho_i$ [31].

threshold as suggested by the results from the local ballooning calculation using the FULL code [36], yielding $\gamma' = -0.2 v_{T_i}/aL_{T_i}$. For the values mentioned above, equation (8) yields $\Delta \simeq 18\rho_i$, in the rough range of fluctuation spreading observed in our simulation, $\Delta \simeq 25\rho_i$. Considering the simplicity of our nonlinear model, this level of agreement is encouraging. If we use the value of Δ from the simulation, equation (10), which is based on our simple one-dimensional theory, yields

$$\chi_i \propto \frac{\chi_{\rm gyroBohm}}{1 + 100\rho^*}, \tag{16}$$

which is in rough agreement with the scaling trend observed in the simulations [9]. Note that the coefficient 100 derived here applies to the particular set of simulation parameters considered here. It could depend on dimensionless variables other than ρ^*. We also note that a similar argument based on equation (10) can also be used as a possible explanation for the previous observation of the 'worse than Bohm transport scaling' from an early global gyrokinetic simulation [7, 26]. In this simulation without zonal flows, the radially global toroidal modes with radial width $\Delta r \propto \sqrt{W\rho_i}$ can easily lead to Bohm scaling of transport in an ideal plasma without the radial spreading of turbulence. In this case, our theoretical results in equation (8) indicate that the width of spreading, $\Delta \propto \sqrt{W\rho_i}$, is due to $\Delta r \propto \sqrt{W\rho_i}$, although there has not been a systematic scaling study of the radial spreading in the absence of zonal flows. Following the same logic, one could deduce that $\chi_i \propto \chi_{\rm Bohm}/(1 + \Delta/W)$ due to the radial spreading of turbulence, and the resulting transport scaling is worse than Bohm. The observation of the radial spreading of fluctuations in this simulation also suggests that while the zonal flows play a crucial role in regulating turbulence and in making linear toroidal coupling weaker, it does not seem to play a direct role in either causing or inhibiting the radial spreading. Another independent global gyrokinetic simulation [6] in the presence of weak zonal flow shear has reported a trend that transport scaling is close to Bohm but tends to change very slowly from Bohm to gyroBohm as the system size gets larger. This intermediate result from a simulation that stands between the simulation of [9] with strong zonal flow shear and the simulation of [7] without zonal flows is also in qualitative agreement with a hypothesis that the radial spreading of turbulence can change the transport scaling by increasing the volume of the active turbulence zone and decreasing the local fluctuation intensity in the region where the instability is linearly strong.

4. Conclusions

In this paper, we have identified and studied in depth the simplest, most minimal problem of turbulence spreading corresponding to the spatio-temporal propagation of a patch of turbulence from a region where it is locally excited to a region of weaker excitation or even local damping. This process is described by a single model equation for the local turbulence intensity, $I(x, t)$, which includes the effects of local linear growth and damping, spatially local nonlinear coupling to dissipation and spatial scattering of turbulence energy induced by nonlinear coupling. The principal results of this paper are as follows:

(i) In the absence of dissipation, front propagation into the linearly stable zone occurs with the property of rapid progression at small t, with a characteristic speed $U_x = 2\epsilon\chi_0/W^2$, followed by slower sub-diffusive progression at late times with $\triangle x \sim (\chi_0 Q_0 t)^{1/3}$.

(ii) The radial spreading of turbulence into the linearly stable zone reduces the turbulence intensity in the linearly unstable zone and introduces an additional dependence on the $\rho^* \equiv \rho_i/a$ to the turbulent intensity, which is otherwise determined by local physics. Since χ_i scales with the turbulent intensity, I, this in turn can cause a change in the transport scaling with respect to ρ^*. These are in broad, semi-quantitative agreement with a number of global gyrokinetic simulation results with strong zonal flows [9], with weak zonal flows [6] and without zonal flows [7].

(iii) The front propagation stops when the radial flux of fluctuation energy from the linearly unstable region is balanced by local dissipation in the linearly stable region. This work provides a new rule of thumb for determining a local fluctuation amplitude in the linearly stable zone.

It is worthwhile to note that the radial spreading of turbulence discussed here could play a role in tokamak plasmas. For instance, large electron thermal diffusivity, χ_e, as observed with the weak gradients in the core of reversed shear plasmas [37–39], is difficult to reconcile with the conventional linear stability based picture in which a nonzero fluctuation level is expected only in the region where the linear growth rate is positive.

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References

[1] Diamond P H and Hahm T S 1995 *Phys. Plasmas* **2** 3640
[2] Itoh S-I and Itoh K 2000 *Plasma Phys. Control. Fusion* **43** 1055
[3] Champeaux S and Diamond P H 2001 *Phys. Lett.* A. **288** 214

[4] Horton W 1999 *Rev. Mod. Phys.* **71** 735
[5] Kadomtsev B B 1965 *Plasma Turbulence* (New York: Academic)
[6] Sydora R D *et al* 1996 *Plasma Phys. Control. Fusion* **38** A281
[7] Parker S *et al* 1996 *Phys. Plasmas* **3** 1959
[8] Garbet X *et al* 1994 *Nucl. Fusion* **34** 963
[9] Lin Z *et al* 2002 *Phys. Rev. Lett.* **88** 195004
[10] Lin Z *et al* 1999 *Phys. Rev. Lett.* **83** 3645
[11] Hahm T S, Diamond P H and Lin Z 2002 Turbulence spreading and tokamak transport scaling *ITPA Transport/ITB Physics Topical Group Third Meeting (Cadarache, France, October 2002)*
[12] Lin Z *et al* 2002 Size scaling of turbulent transport in tokamak plasmas *Paper* TH/1-1 (*IAEA, Lyon, 2002*)
[13] Diamond P H *et al* 1995 *Phys. Plasmas* **2** 3685
[14] Diamond P H *et al* 1997 *Phys. Rev. Lett.* **78** 1472
[15] Lebedev V B and Diamond P H 1997 *Phys. Plasmas* **4** 1087
[16] Barenblatt G I 1979 *Similarity, Self-similarity, and Intermediate Asymptotics* (New York and London: Consultant Bureau)
[17] Itoh S-I and Itoh K 2000 *J. Phys. Soc. Japan* **69** 408
[18] Sarazin Y *et al* 2000 *Phys. Plasmas* **7** 1085
[19] Villard L *et al* 2004 *Nucl. Fusion* **44** 172
[20] Lin Z *et al* 1998 *Science* **281** 1835
[21] Hahm T S *et al* 1999 *Phys. Plasmas* **6** 922
[22] Chen L *et al* 2003 *Phys. Rev. Lett.* submitted
[23] Chen L *et al* 2000 *Phys. Plasmas* **7** 3129
[24] Garbet X 2002 private communication
[25] Kishimoto Y *et al* 1996 *Phys. Plasmas* **3** 1289
[26] Lee W W and Santoro R 1997 *Phys. Plasmas* **4** 169
[27] Diamond P H *et al* 1998 *Plasma Physics and Controlled Nuclear Fusion Research* (Vienna: IAEA) IAEA-CN-69/TH3/1
[28] Hahm T S 2002 *Plasma Phys. Control. Fusion* **44** A87
[29] Idomura Y *et al* 2000 *Phys. Plasmas* **7** 3551
[30] Kim E J *et al* 2003 *Nucl. Fusion* **43** 961
[31] Lin Z and Hahm T S 2004 Turbulence spreading and transport scaling in global gyrokinetic particle simulation *Phys. Plasmas* at press
[32] Biglari H, Diamond P H and Terry P W 1990 *Phys. Fluids* B **2** 1
[33] Hahm T S and Burrell K H 1995 *Phys. Plasmas* **2** 1648
[34] Dimits A M *et al* 2000 *Phys. Plasmas* **7** 969
[35] Hahm T S *et al* 2000 *Plasma Phys. Control. Fusion* **42** A205
[36] Rewoldt G and Tang W M 1990 *Phys. Fluids* B **2** 318
[37] Zarnstorff M 1998 *Bull. Am. Phys. Soc.* **43** 1635
[38] Fujita T *et al* 1997 *Phys. Rev. Lett.* **78** 2377
[39] Stallard B W *et al* 1999 *Phys. Plasmas* **6** 1978