Global gyrokinetic simulation of neoclassical ambipolar electric field and its effects on microturbulence in W7-X stellarator

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ABSTRACT
Global neoclassical simulations of a model equilibrium of the W7-X stellarator find an ambipolar electric field with either an ion root or an electron root solution depending on the temperature ratio between electrons and ions. The ambipolar electric field is then used as an equilibrium field in the turbulence simulations of ion temperature gradient (ITG) instability. The shear of the ambipolar electric field has modest effects on the ITG linear instability, nonlinear saturation amplitude, and turbulent transport in the ion root case. However, in the electron root case, the ambipolar electric field significantly reduces the linear ITG growth rate, turbulence intensity, and radial correlation length, resulting in an ion heat conductivity comparable to the neoclassical transport level in the strong shear region.

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I. INTRODUCTION
Stellarator1–3 is an attractive fusion reactor concept which is designed with minimal plasma current for steady state operations to avoid disruptions. However, the 3D magnetic field in the stellarator greatly enhances the neoclassical (collisional) transport over the level in the axisymmetric tokamak. Recent optimization of the magnetic configuration in the stellarator, e.g., drift orbit optimization in the Wendelstein 7-X (W7-X),4 significantly reduces the neoclassical transport. Turbulent transport then becomes an important issue for the plasma confinement in the stellarator. For example, anomalous transport dominates the particle and heat transport in the outer region of the W7-X stellarator. Density fluctuations measured by the phase-contrast imaging (PCI) in the W7-X exhibit characteristics of the microturbulence, such as ion temperature gradient (ITG) turbulence.5 Therefore, we need to improve our understanding of the properties of the microturbulence in the current experiments and in the design of future stellarators.

Even in the optimized stellarator, neoclassical transport could have significant effects on the turbulent transport. The neoclassical particle fluxes in the stellarator are intrinsically nonambipolar due to the 3D magnetic field, which leads to the formation of an ambipolar radial electric field with two possible solutions of an ion root or an electron root, depending on the temperature ratio between electrons and ions. The shear of this electric field could suppress the microturbulence and possibly provide a bifurcation mechanism to improved plasma confinement,4 as observed experimentally in the W7-AS,16 CHS,11 and LHD12 stellarators. The effects of the electric field shear on the ITG instability in the W7-X and LHD have been studied in linear simulations by a global gyrokinetic code EUTERPE13 using a model electric field as a solution for the ion root. The effects of a constant electric field (i.e., no shear) on nonlinear turbulent transport in W7-X have been studied by a gyrokinetic code GENE.14 While it is expected that a strong electric field could suppress the microturbulence,15 an outstanding issue is whether the self-consistent neoclassical ambipolar electric field is strong enough to suppress the microturbulence in the stellarator.

Gyrokinetic simulation is a powerful tool for studying neoclassical and turbulent transport in the stellarator. Most of the previous nonlinear simulations of microturbulence in the stellarator are local flux-tube simulations,16–18 which have provided useful insights on turbulent transport in the stellarator. The flux-tube simulation19 assumes an axisymmetric equilibrium (i.e., every magnetic field line is equivalent), a finite magnetic shear (i.e., out-going boundary condition in the magnetic field line direction), and a radially translational symmetry.
(i.e., high-n ballooning mode). Recently, global nonlinear simulations of the microturbulence in the stellarator have been carried out using adiabatic electrons by the gyrokinetic codes GTC, XGC-S, and GENE. Global simulation is necessary to incorporate important effects of the 3D magnetic geometry on the neoclassical and turbulent transport, such as the secular radial motion of the helically trapped particles, the localization of drift wave eigenmodes to discrete magnetic fieldlines (i.e., linear coupling of multiple-n toroidal harmonics), the effects of equilibrium sheared flows, the radial variations of turbulence intensity (e.g., turbulence spreading from linear unstable region to stable region), and the linear coupling between zonal flows and low-n harmonics of the microturbulence.

In this paper, we report the first global nonlinear simulation of the microturbulence in the stellarator with the neoclassical ambipolar electric field self-consistently calculated using the GTC. Neoclassical simulation has previously been developed and implemented in the GTC, which was subsequently used for predicting the neoclassical transport and associated ambipolar electric field in the design of a compact stellarator. GTC has recently been applied for simulations of both microturbulence and neoclassical transport in the stellarators and tokamaks with 3D magnetic field.

In this work, GTC neoclassical simulations of a W7-X model equilibrium self-consistently calculate the ambipolar radial electric field, which exhibits either an ion root (in the case of same ion and electron temperatures) or an electron root (in the case of higher electron temperature in the core), in qualitative agreement with other drift kinetic calculations. The ambipolar electric field is then used as an equilibrium field in the GTC turbulence simulations of the ion temperature gradient (ITG) instability. The shear of the ambipolar electric fields has modest effects on the linear ITG growth rate and mode structure in the ion root case, while in the electron root case, significantly reduces the linear ITG growth rate and mostly suppresses the mode amplitude in the strong shear region. Finally, global nonlinear simulations find that the ambipolar electric field has modest effects on the ITG turbulence in the ion root case, where the turbulent transport level is much larger than the neoclassical transport. However, in the electron root case, the ambipolar electric field strongly suppresses the ITG turbulence intensity and reduces the radial correlation length, leading to an ion heat conductivity comparable to the neoclassical transport level in the high shear region. The GTC simulation results qualitatively agree with the transport analysis in a W7-X experiment, which shows that the neoclassical and turbulent heat flux dominates the heat transport in the high and low shear region, respectively.

The paper is organized as follows. The GTC neoclassical and turbulence simulation models for the stellarator are described in Sec. II. Simulations of neoclassical transport and self-consistent ambipolar electric field in the W7-X are presented in Sec. III. Section IV describes simulations of the effects of ambipolar electric field on the ITG turbulence and transport. Finally, conclusions and discussions are discussed in Sec. V.

II. SIMULATION MODEL AND STELLATOR GEOMETRY

There is an important difference between neoclassical and turbulent transport concerning ambipolarity in the stellarator. As demonstrated in gyrokinetic simulations, turbulent transport is automatically ambipolar due to the quasi-neutrality constraint for the microturbulence with the frequency much smaller than the plasma frequency and the wavelength much longer than the Debye length. However, the neoclassical transport is, in general, not ambipolar unless the electric field assumes a particular value. The neoclassical ambipolar electric field $E_a$ can thus be calculated from the neoclassical simulation, without the knowledge of the turbulent transport. The shear of the ambipolar electric field could then affect the microturbulence.

In this section, the simulation model for neoclassical transport and electrostatic turbulence with ambipolar electric field is described in Sec. II A, and the implementation of 3D toroidal geometry, equilibrium profiles, and other simulation settings are described in Sec. II B.

A. Neoclassical and turbulence simulation models

We begin with the gyrokinetic equation describing toroidal plasmas in an inhomogeneous magnetic field,

$$\frac{d}{dt}f(X, \mu, v, t) = \left[ \frac{\partial}{\partial X} + X \cdot \nabla + v \frac{\partial}{\partial v} - C \right] f = 0, \quad (1)$$

where

$$X = v B + v E + v_0,$$

$$v_0 = -\frac{1}{m_a B} \cdot (\mu \nabla B + Z_e e \nabla \phi).$$

Here, the subscript a denotes the particle species (i for ion and e for electron), $f(X, \mu, v, t)$ is the gyrocenter distribution function, X is the gyrocenter position, $\mu$ is the magnetic moment, $v_0$ is the parallel velocity, $t$ is the time, $C$ is the collision operator, $Z_e e$ and $m_a$ the charge and mass, respectively. B is the equilibrium magnetic field, $B^*$ is the magnitude of magnetic field, $X = B + \frac{c_0}{e} \nabla \times b$, $\Omega_a = \frac{c_0 B}{m_a e}$ is the Larmor frequency, $c_0$ is the speed of light, and $b = \frac{B}{B^*}$. The $E \times B$ drift and magnetic drift are $v_E = \frac{\partial X}{\partial c_0}B$, $v_d = v_c + v_E = \mu \nabla \phi + Z_e e \nabla \phi \frac{\partial \phi}{\partial b} + \frac{c_0 v_0 B}{Z_e e B}$, where $b^* = b \cdot B^*$, $v_c$ and $v_d$ are the magnetic curvature and gradient drifts, $\phi = \langle \phi \rangle + \delta \phi + \phi_{nc}$ is the gyroaveraged total electrostatic potential, which includes zonal component $\langle \phi \rangle(\psi)$, nonzonal component $\delta \phi(\psi, \theta, \zeta)$, and neoclassical ambipolar component $\phi_{nc}(\psi)$. Here, the neoclassical ambipolar electric field is dominated by the flux surface-averaged component since plasma rotation is heavily damped by the 3D magnetic geometry of the stellarator. The $E \times B$ drift velocity $v_E$ can be separated into the turbulent part $v_{E\text{ turb}} = \frac{\partial X}{\partial c_0}B$ and the neoclassical part $v_{E\text{ nc}} = \frac{\partial X}{\partial c_0} \phi_{nc}$. To reduce the particle noise, the perturbative $\delta \phi$ method is used in both the turbulence and neoclassical simulations. The distribution function $f_a = f_{0a} + \delta f_a$ can be decomposed into the equilibrium part $f_{0a}$ and the perturbed part $\delta f_a$. The propagator in Eq. (1) is also separated into the equilibrium part $L_0$ and the perturbed part $\delta L$. Equation (1) can then be written as $(L_0 + \delta L)(f_{0a} + \delta f_a) = 0$. The equilibrium distribution function $f_{0a}$ is defined by

$$L_0 f_{0a} = 0. \quad (2)$$

The perturbed distribution function is dynamically calculated by

$$\langle L_0 + \delta L \rangle \delta f_a = -\delta L f_{0a}. \quad (3)$$
The propagators and equilibrium distribution function are defined differently for the neoclassical simulation model and for the turbulence simulation model.

For the neoclassical simulation with collisions, the propagators $L_0$ and $\delta L$ are defined as

$$L_0 = \frac{\partial}{\partial t} + \nu_b \cdot \nabla - \frac{1}{m_0} \frac{b \cdot \mu \nabla B}{B} \frac{\partial}{\partial \nabla \parallel} - C(f_{\text{eq}}), \quad (4)$$

$$\delta L = (v_d + v_{\text{eq}}) \cdot \nabla - \frac{1}{m_0} \frac{v_f}{\Omega_{\text{gyr}}} \nabla \times b \cdot \mu \nabla B + \frac{B'}{B} Ze \nabla \phi_{\text{nc}} \nabla \parallel \frac{\partial}{\partial \nabla \parallel} - C(\delta f_s). \quad (5)$$

Both ion and electron are treated as drift kinetic particles in the neoclassical simulation. The Fokker–Plank collision operator, which conserves energy and momentum, is used for like-species collisions for ion and electron species, and pitch angle collision operator is used for electron–ion collisions. The equilibrium distribution function $f_{\text{eq}}$ for the neoclassical simulation is a local Maxwellian, which is an exact solution of Eq. (2) since the guiding center drifts are not included in the turbulence propagator $L_0$ defined in Eq. (4). Combining the gyrokinetic Poison’s equation and guiding center continuity equation, the ambipolar radial electric field $E_r$ is calculated from the equation,

$$\frac{dE_r}{dt} = \frac{\nabla \Psi^2}{B^2} = -\frac{1}{m_n n_c} \sum Z_n \Psi \left[ d^2 \nabla f_{\text{nc}}(v_d + v_{\text{eq}}) \cdot \nabla \Psi \right]. \quad (6)$$

where the angle bracket $\langle \ldots \rangle$ represents the flux-surface average, the symbol $[\ldots]$ means taking the amplitude of a vector, and $v$ is the velocity. Here we define $E_r \equiv -\frac{\partial}{\partial \nabla \parallel} \Psi$ as the poloidal magnetic flux, and the relation between $E_r$ and radial electric field $E_r = E_r + \nabla \Psi$. The complete description of this neoclassical simulation model with the $E_r$ can be found in Ref. 29.

For the collisionless turbulence simulation, the propagators $L_0$ and $\delta L$ are defined as

$$L_0 = \frac{\partial}{\partial t} + (v_d b_0 + v_d + v_{\text{eq}}) \cdot \nabla - \frac{1}{m_0 B^2} \left( \mu \nabla B + Ze \nabla \phi_{\text{nc}} \right) \frac{\partial}{\partial \nabla \parallel}, \quad (7)$$

$$\delta L = v_{\text{turb}} \cdot \nabla - \frac{1}{m_0 B^2} Ze \nabla \phi_{\text{nc}} \delta f_s \frac{\partial}{\partial \nabla \parallel}. \quad (8)$$

In this turbulence simulation model, we assume gyrokinetic ions and adiabatic electrons. The equilibrium distribution function $f_{\text{eq}}$ for turbulence simulation is the neoclassical solution from Eqs. (1)–(6). However, an analytic form of $f_{\text{eq}}$ is not generally available. Similar to most of the gyrokinetic turbulence simulations, we approximate $f_{\text{eq}}$ in the right-hand side of the perturbed distribution function [Eq. (3)] as a local Maxwellian. We also assume that the equilibrium parallel flow is strongly damped in the W7-X stellarator. The zonal and nonzonal components of electrostatic potentials ($\phi$) and $\delta \phi$ are solved using the quasi-neutrality condition, and the neoclassical component $\phi_{\text{nc}}$ is obtained from the neoclassical simulations and considered as a static equilibrium potential in the turbulence simulation. More complete electrostatic simulation model for the stellarator is described in Ref. 21. Here, we extend this turbulence simulation model by incorporating the neoclassical ambipolar electric field $E_r$ as a part of the equilibrium.

In both neoclassical and turbulence simulations, the gyrocenter motion follows the canonical equations in Boozer coordinates. The perturbed propagator $\delta L$ [Eq. (8)] is removed on the left-hand side of the perturbed distribution function [Eq. (3)] in the linear simulation of the ITG instability, while it is kept in the nonlinear simulation of the ITG turbulence. The neoclassical ambipolar potential $\phi_{\text{nc}}$ is considered as part of the equilibrium for the turbulence simulation, and thus kept in both the linear and nonlinear turbulence simulations.

As the first step toward self-consistent simulation of interaction between neoclassical and turbulent transport, we perform simulations of neoclassical transport and turbulent transport separately. The neoclassical-ambipolar radial electric field calculated in the neoclassical simulation is then used as part of the equilibrium for the turbulence simulation. In future work, we will perform self-consistent simulations coupling neoclassical and turbulent transport.

### B. Stellarator geometry

In the current simulation of the neoclassical transport and ITG turbulence in the W7-X stellarator, GTC uses nonaxisymmetric equilibrium constructed by the ideal MHD code VMEC, which assumes closed magnetic surfaces. The equilibrium geometry and magnetic field are provided by VMEC as the Fourier series in both the poloidal and toroidal directions on a discrete radial mesh with equidistant in the toroidal flux. These 3D data in the left-handed straight-fieldline coordinates are then transformed to the Boozer coordinates as the Fourier series in the toroidal direction on discrete grid points on the 2D poloidal plane

$$B(\psi, \theta, \zeta) = \sum_n \left[ B_1(\psi, \theta, n) \cos(n \zeta) + B_2(\psi, \theta, n) \sin(n \zeta) \right], \quad (9)$$

where $B_1$ and $B_2$ are the coefficients of the Fourier series, $n$ is the toroidal harmonic, $(\psi, \theta, \zeta)$ are the poloidal magnetic flux, poloidal angle, and toroidal angle, respectively, which form the right-handed Boozer coordinates. The equilibrium geometry in the Boozer coordinates can be transformed to the cylindrical coordinates $(R, \phi, Z)$. Here, $R$ and $Z$ are Cartesian coordinates on the poloidal plane, and $\Phi$ is the toroidal angle in the cylindrical coordinates. The position in $R, Z$ and difference between $\Phi$ and $\zeta$ are all given by the Fourier series similar to Eq. (9). The equilibrium 3D magnetic field and metric tensor are represented in the GTC by a 3D quadratic spline interpolation on an equilibrium mesh. On the other hand, the fluctuating quantities, such as the electrostatic potential, are represented in a global field-aligned mesh in a general toroidal geometry, which provides the maximal computational efficiency without making any geometry approximation.

The magnetic geometry of the W7-X stellarator used in this work is a modified equilibrium adopted from Refs. 13 and 21. The W7-X has a stellarator symmetry with a field period of $N_B = 5$, which means all the equilibrium quantities have a periodicity of $2\pi/N_B$ in the toroidal direction, as shown in Fig. 1(a). Therefore, we can construct the equilibrium mesh in the domain $\zeta = \left[ 0, 2\pi/N_B \right]$, with the toroidal periodicity. This field period is used in the turbulence simulation in Sec. IV. For neoclassical transport, simulations using either one-fifth of the torus or the full torus are equivalent. For turbulent transport, there are five drift wave eigenmode families corresponding to the five field.
Depending on the temperature ratio between ions and electrons, there could be two solutions for the neoclassical ambipolar electric field $E_i$: an electron root with $E_i > 0$, and an ion root with $E_i < 0$. We have carried out two cases with different electron temperatures. One simulation case with $T_i = T_e = T = 1 \text{ keV}$ has an ion root solution for the ambipolar electric field. Another simulation case with $T_i = T_1$ and $T_e = T_2 \sim 5 \text{ keV}$, which has an electron root solution due to the high electron temperature in the core, has similar temperature profiles in some W7-X experiments with electron cyclotron resonance heating (ECRH). Note that the pressure profiles and the magnetic geometry are not fully consistent. Since all simulations presented in this manuscript are electrostatic simulations using the perturbative method (of method), we can use various pressure profiles (e.g., ion root and electron root) for the same magnetic geometry, as is done in many gyrokinetic codes.

GTC uses three meshes: equilibrium mesh, neoclassical mesh, and turbulence mesh. The equilibrium quantities are represented on the equilibrium mesh, where the radial, poloidal, and toroidal grid numbers are, respectively, 200, 799, and 51 over one field period (i.e., one-fifth of the whole torus). In the neoclassical simulation, the momentum and energy conservation in the Fokker–Plank collision operator are enforced on a neoclassical mesh, where the radial, poloidal, and toroidal grid numbers are, respectively, 64, 64, and 32 based on the convergence studies. The neoclassical ambipolar electric fields are averaged over the flux surface and thus represented on the turbulence mesh. We use both the drift kinetic ion and electron in the neoclassical simulations with realistic electron mass and ion mass. Based on the convergence studies, the particle number is $2.3 \times 10^6$ for both the ion and electron. The effective collision frequency is defined as $\nu' = \nu = \frac{3}{2} \nu v_{th} / v_p$, where $\nu = r / R_0$ is the local inverse aspect ratio, $r$ is the radius evaluated on the outer midplane at $\zeta = 0$, $R_0 = 5.62 \text{ m}$ is the mean major radius over $\zeta = [0, 2\pi]$, $\nu$ is the physical collision frequency, and $v_{th} = \sqrt{T_{ei}/m_i}$ is the thermal velocity of the ion or electron. Both the ion root and electron root cases are in the long mean free path regime. For the ion root case, $\nu'\psi = 0.5 \nu \psi$ is 0.64 for the ion, and 0.75 for the electron. For the electron root case, $\nu'\psi = 0.5 \nu \psi$ is 0.64 for the ion, and 0.43 for the electron. Here $\psi$ is the value of $\psi$ at the separatrix. The time step size in the simulation is $\Delta t = 0.00025 R_0 / C_i$, where $C_i = \sqrt{\nu / m_i}$ is the speed of the ion-acoustic wave.

In most of the linear ITG simulations, one-fifth of the W7-X torus is used with toroidal periodicity. Based on convergence studies, the radial, poloidal, and parallel grid numbers are, respectively, 128, 2200, and 9 over one-fifth of the torus in the turbulence mesh for the perturbed electrostatic potentials. The only exception is the electron root case, which requires 256 radial grids for numerical convergence. The turbulence mesh is the global field-aligned mesh, which allows us to use a small parallel grid number since the ITG mode has $k \parallel \ll k_z$. The nine parallel grids over one-fifth of the torus correspond to about 50 grids per parallel wavelength since the ITG mode parallel wavelength is on the order of the connection length ($2\pi a / R_0$). Without the global field-aligned mesh, the toroidal grid number would have to be about 2000 (i.e., 220 times more expensive) to achieve the same resolution. The number of particles per cell is 50, and the time step size is $\Delta t = 0.01 R_0 / C_i$. The time step size convergence has been studied in earlier GTC simulations of the ITG turbulence in the same W7-X equilibrium. In the nonlinear ITG simulations, the number of

FIG. 1. (a) Real space 3D plot of magnetic field $B$ on $\psi = 0.5 \psi_w$ flux surface of W7-X. (b) Radial profiles of equilibrium density, temperature, and rotational transform $\tilde{\iota}$. Ion root case has $T_i = T_e = T$ and electron root case has $T_i = T_i$, $T_e = T_2$. The simulation domain is marked by two magenta lines: $\psi_{inner} = 0.2 \psi_w$ and $\psi_{outer} = 0.8 \psi_w$.
particles is four times larger than the linear simulations to achieve the numerical convergence, and the time step size and the grid points are the same as the linear simulations.

The region with a strong pressure gradient, \( \dot{\psi} = [0.2, 0.8] \), is chosen as the simulation domain for both the neoclassical and turbulent transport. In the buffer zone (\( \dot{\psi} < 0.3 \) or \( \dot{\psi} > 0.7 \)), equilibrium density and temperature gradients are set to be zero and the perturbed electrostatic potential is smoothly suppressed to be zero at the radial boundaries. When a particle goes outside the radial boundary in the simulation, it is put back to the simulation domain and its weight is set to zero. So the particles that go outside the radial simulation domain will not affect the self-consistent electrostatic potential, but the total marker number is kept constant.

III. SIMULATION OF NEOCLASSICAL AMBIPOLAR ELECTRIC FIELD IN W7-X

A. Neoclassical ambipolar electric field

In GTC neoclassical simulations solving Eqs. (1)–(6), the radial electric field is initially set to zero. Radial current is subsequently generated by the difference between ion and electron particle fluxes

\[
\Gamma_a = \left\langle \int d^3 \nu d f_a (\nu_d + \nu_i) \cdot \nabla \psi \right\rangle,
\]

which induced a radial electric field \( E_r \) to maintain neutrality. Time history of the volume-averaged \( \Gamma_a \) in the ion and electron root cases are shown in Fig. 2(a). After about half of the ion collision time \( \tau_i = 1/\nu_i \), ambipolar state is achieved with the steady state \( E_r \) in both ion and electron root cases. Even though the \( \Gamma_a \) has some fluctuation in the time evolution, the \( E_r \) shown in Fig. 2(b) is the time integration of \( \Gamma_a \), thus it is much smoother. During the \( E_r \) evolution, the geodesic acoustic mode (GAM) oscillation is not visible, because of the large damping rate of the GAM due to the low safety factor \( q = \frac{1}{\tau_i} \approx 1.1 \) in the W7-X.44

Figure 2(c) shows the neoclassical ambipolar electric field \( E_r \) at the steady state in the neoclassical simulations of the ion and electron root cases. The ion root case has a relatively flat ambipolar electric field with \( E_r < 0 \) across the whole simulation domain. In contrast, the electron root case has a much stronger ambipolar electric field with \( E_r > 0 \) in the inner region, and \( E_r < 0 \) in the outer region, which creates a strong \( E \times B \) flow shear around the \( \dot{\psi} \sim 0.5 \) flux surface. This \( E_r \) profile of the electron root agrees qualitatively with other neoclassical codes [e.g., Drift Kinetic Equation Solver (DKES)] and measurements in some W7-X ECRH experiments with high electron temperature,\(^{43}\) with large positive \( E_r \) values transitioning to negative \( E_r \) values with an inflection point around \( \dot{\psi} = 0.5 \). Note that the equilibrium density and temperature profiles we use in the simulations are only qualitatively similar to the experimental profiles, so the agreement of the \( E_r \) profile between simulation and experiment is expected to be qualitative.

B. Neoclassical transport with self-consistent ambipolar electric field

The irreducible neoclassical transport provides a reference level for the turbulent transport in the stellarator. Neoclassical theory predicts that diffusivities are inversely proportional to collision frequency due to the helical trapped particles in the long mean free path regime of the stellarator,\(^{45}\) which is different from the axisymmetric tokamak. With the neoclassical ambipolar electric field \( E_r \), the diffusivities would...
decrease if $E_r$ is large enough. In the GTC neoclassical simulations, heat conductivities are calculated as

$$Z_s = \frac{1}{\langle \nabla \psi \rangle m_0} \left\langle \int d^3 \nu \delta f_\nu \left( \frac{1}{2} m_\nu v^2 - \frac{3}{2} T_\nu \right) \nu_\nu \cdot \nabla \psi \right\rangle$$

and diffusivities as

$$D_\nu = \frac{1}{\langle \nabla \psi \rangle m_0} \left\langle \int d^3 \nu \delta f_\nu \nu_\nu \cdot \nabla \psi \right\rangle.$$

We compare the diffusivities and heat conductivities of ion ($D_i$, $\chi_i$) and electron ($D_e$, $\chi_e$) averaged over $t = [0.75, 1.0] \tau_i$, between simulations with or without solving the self-consistent ambipolar electric field $E_r$ in Figs. 3 and 4, where the gyro-Bohm unit $D_{GB} = \chi_{GB} = \frac{2}{3} \nu_i^3$ are used for normalization, with $\nu_i = \sqrt{\frac{T_i}{m_i}}$ as thermal speed of ion, $a = 0.08$ $R_0$ as minor radius on the outer midplane at $\zeta = 0$, $\rho_i = \nu_i m_i c/eB$ as ion Larmor radius. The results in the buffer zone of simulations near the boundary are not physical and not plotted. The diffusivities are shown in Figs. 3(a) and 4(a). In the ion root case without the $E_r$, the $D_i$ is typically one order of magnitude larger than the $D_e$ in the whole simulation domain. In the electron root case, the $D_e$ is about three times of the $D_i$ in the region near $\psi \sim 0.5$, where the electrons have lower effective collision frequency than the ions. In the simulation with self-consistent ambipolar electric field $E_r$, both the ion root case and electron root case have $D_i \approx D_e$ in the whole simulation domain, which ensures the ambipolarity when the $E_r$ approaches a steady state solution.

The heat conductivities are shown in Figs. 3(b) and 4(b). In the ion root case without the ambipolar electric field $E_r$, the $\chi_i$ is one order of magnitude larger than the $\chi_e$ in the region near $\psi \sim 0.5$. With the $E_r$, the $\chi_e$ changes little, but the $\chi_i$ decreases to a level close to the $\chi_e$. In the electron root case without the $E_r$, the $\chi_e$ is about two times of the $\chi_i$. With the $E_r$, the $\chi_e$ decreases by a factor of four and the $\chi_i$ by a factor of two. These transport coefficients are mostly due to the helical trapped particles and agree with the neoclassical theory. The
effects of ambipolar electric field

In this subsection, we perform nonlinear simulations to study the effects of ambipolar electric field $E_{eq}$ on the ITG turbulence in the W7-

X. Both the ion root and electron root cases are studied using the equilibrium density, temperature, and neoclassical ambipolar electric field $E_{eq} = -\frac{\partial \phi_{eq}}{\partial \psi}$ same as in the linear simulations. The nonlinear terms are added to the gyrocenter motion and the zonal electric field $E_{ZF} = -\frac{\partial \phi}{\partial \psi}$ generated by the turbulence is also included in the simulations, since it has been shown\(^\text{11}\) that the zonal flows are the dominant saturation mechanism for the ITG instability in the W7-X. To prevent the temperature profile relaxation, we use an effective collision operator for energy diffusion to model a heat bath,\(^\text{17}\) which maintains the initial ion temperature profile on a scale much longer than the turbulence correlation length.

Figure 7 shows the history of the volume-averaged ion heat conductivity $\chi_i$, root mean square of the nonzonal electrostatic potential $\delta \phi_{rms}$ and zonal flow $\langle \phi \rangle$ from GTC nonlinear simulations with or without the ambipolar electric field $E_{eq}$. The heat conductivities are
calculated as \( \chi_s = \langle (\delta \phi_{\text{ion}}) \rangle_{E_{\text{eq}}} \langle d^\nu \delta f_s \langle \frac{1}{2} m_s v^2 - \frac{1}{2} T_s \rangle \rangle \) in the turbulence simulations. In the ion root case with the \( E_{\text{eq}} \) shown in panels (a) and (c), the saturated amplitude of the \( \chi_s \) and \( \delta \phi_{\text{ion}} \) are very close to that from simulations without the \( E_{\text{eq}} \). Therefore, the effect of the ambipolar electric field on the ITG turbulence and transport is weak in the ion root case. In contrast, the effects of the ambipolar electric field \( E_{\text{eq}} \) on the ITG turbulence and transport are very strong in the electron root case as shown in panels (b) and (d). Compared with simulations without \( E_{\text{eq}} \), the saturated amplitudes of the \( \chi_s \) and \( \delta \phi_{\text{ion}} \) decrease by a factor of 3.5 and 2, respectively. In both the ion root case and the electron root case, the saturated zonal flow \( \langle \phi \rangle \) amplitude in the simulation without the \( E_{\text{eq}} \) is close to that with \( E_{\text{eq}} \), which means the zonal flow amplitude is only slightly affected by the \( E_{\text{eq}} \). However, the shear of the zonal flow can be affected by \( E_{\text{eq}} \), as shown in Fig. 9.

To verify that the simulation results are not affected by particle noise, all of the GTC simulations in this paper use a quasilinear theory together with the measured noise spectrum \(^4\) to monitor the effects of the particle noise. As an example, the noise-driven heat conductivity for ion \( \chi_{\text{noise}} \) in the ion root case without \( E_{\text{eq}} \) is shown in Fig. 7(a). During the whole simulation period, the noise-driven transport is shown to be less than 1% of the ITG-driven heat transport.

Figure 8 shows the radial profiles of the heat conductivity \( \chi_s \) after ITG saturation and averaged over \( t = [15, 25] R_0 / C_s \) for the ion root case, and \( t = [70, 80] R_0 / C_s \) for the electron root case. In the ion root case, effects of the ambipolar electric field on the \( \chi_s \) profiles are insignificant. The \( \chi_s \) driven by the ITG turbulence is much larger than the neoclassical ion heat conductivity \( \chi_{\text{neo}} \) shown in Fig. 3(b). In contrast, the shear of the ambipolar electric field in the electron root case greatly suppresses the \( \chi_s \), especially in the high shear region of the \( E_{\text{eq}} \), which leads to two \( \chi_s \) peaks at \( \psi = 0.38 \) and \( \psi = 0.58 \). In the high shear region near \( \psi = 0.5 \), local transport is mostly driven by turbulent spreading from the low shear region. Compared to the neoclassical ion heat conductivity \( \chi_{\text{neo}} \) shown in Fig. 4(b), the \( \chi_s \) driven by the ITG turbulence in the low shear region is still one order of magnitude larger.
than the $L_{iso}$, but comparable to the $L_{neo}$ in the high shear region near the $\psi = 0.5$ surface.

The GTC simulation results for the turbulent transport qualitatively agree with the transport analysis based on the power balance in a W7-X experiment with ECRH, which shows that the neoclassical heat flux accounts for over 50% of the total heat flux in the high shear region and that the anomalous transport dominates the heat flux in the low shear region.

Both the neoclassical ambipolar electric field and zonal flows\(^5\) reduce the ITG turbulence intensity and transport. The radial profiles of the zonal flow electric field $E_{ZF}$ after the ITG saturation (at $t = 25R_0/C_i$ for the ion root case and $t = 80R_0/C_i$ for the electron root case) and the neoclassical ambipolar electric field $E_{eq}$ are shown in Fig. 9. In the ion root case shown in panel (a), the amplitude and structure of the $E_{ZF}$ are not significantly affected by the $E_{eq}$, and the zonal flow shearing rate is one order of magnitude larger than that of the $E_{eq}$. On the other hand, in the electron root case shown in panel (b), the amplitudes of the $E_{ZF}$ are similar between the simulations with or without the $E_{eq}$. However, the radial wavelength of the $E_{ZF}$ in the simulation with the $E_{eq}$ is much larger than that in the simulation without the $E_{ZF}$. Consequently, the mean shearing rate of the $E_{ZF}$ decreases by a factor of three by the effects of the $E_{eq}$, and is much smaller than that of the $E_{eq}$, possibly due to both the reduction of the ITG turbulence intensity and the enhancement of zonal flow residual level\(^4\) by the $E_{eq}$.

Figure 10 shows the perturbed nonzonal electrostatic potential $\delta \phi$ on the $\zeta = 0$ poloidal plane after ITG saturations (at $t = 25R_0/C_i$ for the ion root case and $t = 80R_0/C_i$ for the electron root case) from simulations with or without the ambipolar electric field $E_{eq}$. In the ion root case, the poloidal snapshot of the $\delta \phi$ without the $E_{eq}$ is very similar to that with the $E_{eq}$, which indicates that the ITG turbulence is not significantly affected by the $E_{eq}$. In contrast, in the electron root case, the large shearing rate of the $E_{eq}$ in the region near the $\psi = 0.5$ flux surface strongly suppresses the local ITG turbulence, resulting in the appearance of two rings in the $\delta \phi$ poloidal snapshot [panel (d)].

The ambipolar electric field $E_{eq}$ can also affect the radial correlation length $L_r$ of the ITG turbulence, which is another important physical quantity measured in the experiments.\(^{26,51}\) Here, we calculate the $L_r$ in both the ion root case and the electron root case after nonlinear ITG saturation.

We define the correlation function $C_r(\Delta r)$ as $C_r(\Delta r) = \frac{\langle \delta \phi(r+\Delta r) \delta \phi(r) \rangle}{\sqrt{\langle \delta \phi^2(r+\Delta r) \rangle \langle \delta \phi^2(r) \rangle}}$, where $r$ is the radial position, $\Delta r$ represents the radial separation, $\langle \cdots \rangle$ represents the average over $t = [15, 25]R_0/C_i$ for the ion root case, and $t = [70, 80]R_0/C_i$ for the electron root case, i.e., about ten ion transit time, $\psi = [0.3, 0.7], \theta = 0$ and $\zeta = [0, 2/5 \pi]$. Here we neglect the separation in the parallel direction, because of the small parallel wave vector $k_z$.\(^21\) The radial correlation length $L_r$ is defined as the $e$-folding decay of the radial correlation function $C_r(L_r) = 1/e$, where $e$ is the natural constant.

The radial correlation functions $C_r(\Delta r)$ in the ion root case and the electron root case are shown in Fig. 11. Both simulations with and without ambipolar electric field $E_{eq}$ are compared together with the
fitting curves of an exponential decay $\exp\left(-\frac{t}{\tau}\right)$. For the ion root case, the ambipolar electric field $E_{eq}$ does not significantly affect the radial correlation length $L_r$. However, in the electron root case, the ambipolar electric field $E_{eq}$ induces a 27% reduction of the radial correlation length $L_r$, from $2.2\rho_i$ to $1.6\rho_i$. The effects of the ambipolar electric field $E_{eq}$ on the radial correlation length $L_r$ is consistent with its effects on the ITG turbulence intensity and the ion heat conductivity.

V. DISCUSSION AND CONCLUSION

In this paper, we use GTC neoclassical simulations to self-consistently calculate the neoclassical ambipolar electric field in a model equilibrium of the W7-X stellarator for both the ion root case and electron root case, in qualitative agreement with other drift kinetic calculations. These ambipolar electric fields are then used as an equilibrium field in the GTC turbulence simulations of ITG instability. In the ion root case, the shear of the ambipolar electric field has modest effects on the linear ITG growth rate and mode structure. On the other hand, in the electron root case, the shear of the ambipolar electric field significantly reduces the linear ITG growth rate and mostly suppresses the mode structure in the high shear region. Finally, the global non-linear simulations show that the shear ambipolar electric field has little effect on the ITG turbulence and transport in the ion root case, and the turbulent transport is much larger than the neoclassical value. In contrast, in the electron root case similar to the W7-X experiments with ECRH, the effects of ambipolar electric field strongly reduce the ITG turbulence intensity and reduce the correlation length, resulting in the ion heat conductivity comparable to the neoclassical transport level in the high shear region.

The finding that the ambipolar electric field of the electron root case significantly suppresses the ITG turbulence and transport could have important implications for future stellarator reactors. In the burning plasmas, thermal electrons will be heated first by energetic fusion products (i.e., $\alpha$-particle) and then the electrons will transfer
their energy to the thermal ions resulting in an electron temperature higher than the ion temperature.

In future turbulence simulations, we will include the effects of kinetic electrons (especially roles of helically trapped electrons in the stellarator), which could enhance ITG instability and excited new trapped electron mode. We will also develop coupled neoclassical and turbulence simulations, where both neoclassical transport and turbulence transport are treated on the same footing. On one hand, neoclassical transport (including ambipolar electric field) could be affected by the microturbulence acting as an effective collision in the particle phase space. On the other hand, Coulomb collisions could damp zonal flows that regulate the microturbulence, and neoclassical simulation is needed to provide an accurate equilibrium distribution function for the turbulence simulation.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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