

Linear properties of reversed shear Alfvén eigenmodes in the DIII-D tokamak

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Abstract

Linear properties of the reverse shear Alfvén eigenmode (RSAE) in a well-diagnosed DIII-D tokamak experiment (discharge #142111) are studied in gyrokinetic particle simulations. Simulations find that a weakly damped RSAE exists due to toroidal coupling and other geometric effects. The mode is driven unstable by density gradients of fast ions from neutral beam injection. Various damping and driving mechanisms are identified and measured in the simulations. Accurate damping and growth rate calculation requires a non-perturbative, fully self-consistent simulation to calculate the true mode structure. The mode structure has no up–down symmetry mainly due to the radial symmetry breaking by the density gradients of the fast ions, as measured in the experiment by electron cyclotron emission imaging. The RSAE frequency up-sweeping and the mode transition from RSAE to TAE (toroidal Alfvén eigenmode) are in good agreement with the experimental results when the values of the minimum safety factor are scanned in gyrokinetic simulations.

1. Introduction

Commonly seen in reversed shear tokamaks [1–6] and localized near the flux surface of the minimum safety factor ($q_{\min}$), reversed shear Alfvén eigenmodes (RSAEs) (also known as Alfvén cascade due to its frequency sweeping phenomenon) [7] can be excited by fast ions and can cause fast ion losses. RSAEs in the local linear ideal MHD limit in simple geometries are well understood [7–13]. Some RSAE kinetic effects are also studied analytically [14, 15] and numerically [16]. Comprehensive studies incorporating global effects, kinetic effects, nonlinear effects and so on require a global, fully self-consistent and nonlinear kinetic simulation. The global gyrokinetic toroidal code (GTC) [17, 18] has been successfully applied to linear simulations of the RSAE in a simple tokamak equilibrium [19].

In this work, the GTC is applied to simulating DIII-D discharge #142111 [20–22], in which RSAE is one of the most significant instabilities driven by energetic particles. In this discharge, the RSAE poloidal mode structure is measured by two-dimensional electron cyclotron emission imaging (ECEI) [20]. Coherent fast ion losses by RSAEs are measured by a fast ion loss detector (FILD) [21, 22]. Here we focus on simulating the $n = 3$ mode in this discharge, which starts as an RSAE at about 700 ms. The frequency sweeps up as $q_{\min}$ decreases.

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Near 750 ms, the frequency gradually stops up-sweeping and the mode makes a transition to two toroidal Alfvén eigenmodes (TAEs).

The recent general geometry upgrade [23] of the electromagnetic version of GTC [18, 19, 24–26] enables simulation with realistic device geometry and equilibrium plasma profiles from the DIII-D experiment. A complete gyrokinetic formulation with equilibrium current [26] in GTC enables comprehensive simulation of kinetic magnetohydrodynamic (MHD) processes driven by both pressure gradients and equilibrium current. In the linear and long wavelength limit, the gyrokinetic model is shown [26] to reduce to ideal MHD theory in the case with single ion species. When fast ions are present, the gyrokinetic formulation is shown [27] to be consistent with hybrid MHD-gyrokinetic models, including the pressure coupling model [28–30] and the current coupling model [28, 31].

Our GTC simulations of the DIII-D discharge #142111 show that RSAE can exist even with the equilibrium current and without the fast ion drive, due to the toroidal coupling and other geometric effects. Various damping and driving mechanisms of the mode are identified and measured, including continuum damping [32], radiative damping [33], ion Landau damping, electron Landau damping, and driving by pressure gradients of both thermal and energetic particles. Our simulation results show that accurate damping and growth rate calculation

requires a non-perturbative and fully self-consistent simulation to calculate the true mode structure. The mode structure has no up-down symmetry due to the radial symmetry breaking by fast ions, as observed in DIII-D experiments and TAEFL [20] and GTC [19] simulations. The frequency up-sweeping of the RSAE and the mode transition from RSAE to TAE are found to be very close to the experimental results when $q_{\min}$ values in our simulations are scanned. Good agreement is also obtained in comparisons of frequencies, growth rates and mode structures among simulation results from GTC, GYRO and TAEFL codes [34].

This paper is organized as follows: first, the simulation setup is given in section 2; then the damping and driving mechanisms are discussed in section 3; the results from scanning $q_{\min}$ including RSAE frequency up-sweeping and RSAE to TAE transition are presented in section 4; the effects of the equilibrium magnetic field and current direction are shown in section 5, and section 6 summarizes this work.

2. Simulation setup

Using the general geometry feature [23] in GTC, fully self-consistent equilibrium geometry and profiles, including the flux surface structure, the field magnitude, and the q -profile, the density and temperature profiles of all three species, i.e. the electron, the background ion and the fast ion, are loaded from the experimental data constructed by EFIT [35] and ONETWO [36]. The equilibrium geometry and radial profiles are shown in figure 1. The major radius on the magnetic axis is $R_0 = 173.86$ cm, which is about 576 times the on-axis ion gyro-radius. The on-axis magnetic field amplitude is $B_a = 20\,145$ G. The $q_{\min}$ surface is at $\rho = 0.33$ where ρ is the square root of the normalized toroidal flux:

$$\rho = \sqrt{\frac{\psi_t}{\psi_{tw}}}, \quad (1)$$

with ψ_t being the toroidal flux and ψ_{tw} being the toroidal flux at the outermost flux surface. Before simulating different $q_{\min}$ cases, we focus on the case with $q_{\min} = 3.18$. Both ion species are deuterium nuclei. In DIII-D discharge #142111, from the top-view of the tokamak, the toroidal magnetic field direction is clockwise, and the toroidal equilibrium current direction is counterclockwise. The simulations presented in sections 3 and 4 are performed with the same magnetic field direction as the experiment, but with the equilibrium current direction opposite to the experiment. However, it is shown in section 5 that changing the equilibrium current direction does not affect the results. Our simulation domain is $0.1 \leq \rho \leq 0.9$. The boundary condition is zero for all perturbed quantities, which is equivalent to a perfect conducting wall. The numerical parameters are chosen based on convergence tests. The converged parameters are used for production runs and they are listed as follows. The number of grid points in the real space is 32 in the parallel direction and 144 in the radial direction. The number of grid points in the poloidal direction depends on the radial position, which keeps the grid size approximately constant. At the $q_{\min}$ flux surface, there are 434 grid points in the poloidal direction. As a result, the real space grid size is about $a\Delta\rho/\rho_i \simeq a\rho\Delta\theta/\rho_i \simeq 1.3$, where a is the outboard midplane minor radius, and ρ_i is the

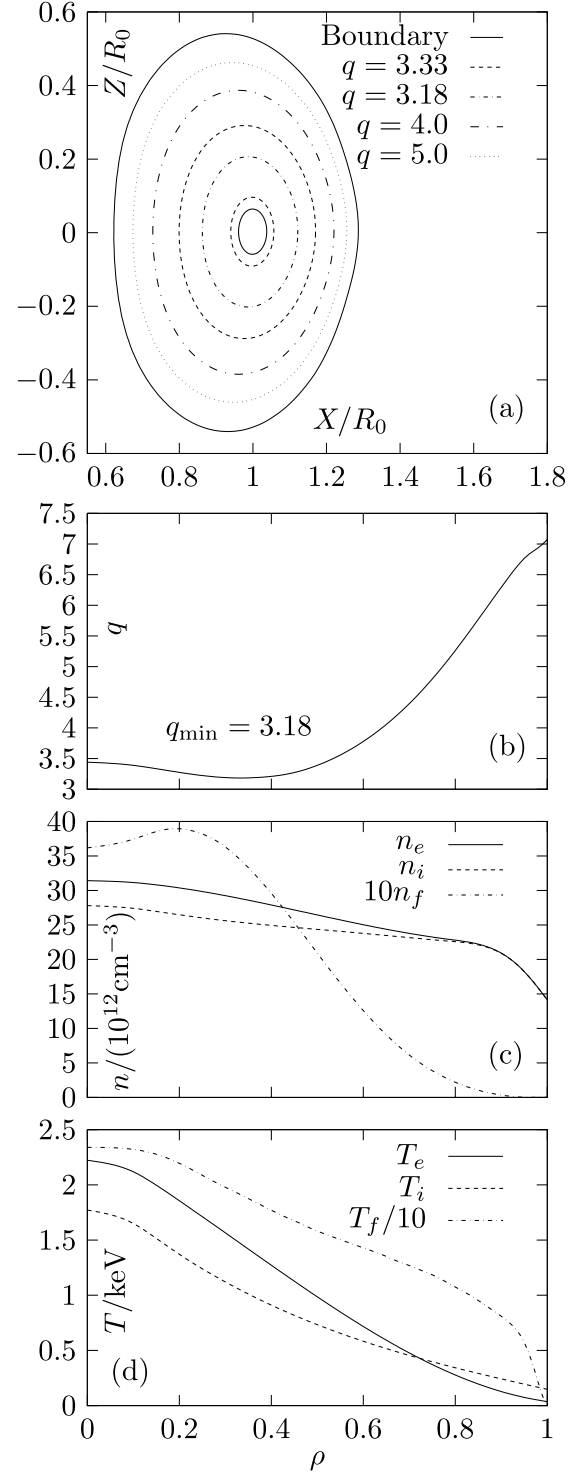


Figure 1. Equilibrium geometry and radial profiles in DIII-D discharge #142111 at 745 ms: (a) flux surface structure in the poloidal plane, (b) q -profile, (c) density profiles, (d) temperature profiles. Data provided by M Van Zeeland [20–22].

thermal ion gyro-radius at the $q_{\min}$ surface. The time step size is $\Delta t = 0.068 v_{A0}/R_0$, where v_{A0} is the on-axis Alfvén speed. Each of the three particle species has 50 marker particles per cell, totalling about 134 million particles for each species. The initial distribution of the marker particles is uniform in real space and Maxwellian in velocity space, cutting off at four

Table 1. Various simulation cases to measure the damping and driving mechanisms of the $n = 3$ mode with $q_{\min} = 3.18$. Three significant digits are consistently kept in this table, but measurement error, finite simulation time, etc can incur up to a few per cent uncertainty.

Case	Description	$(\omega_r, \gamma)/(v_{A0}/R_0)$ $(\omega_r, \gamma)/(2\pi)/\text{kHz}$	Damping and/or driving mechanisms
(I)	Zero temperature ideal MHD	(0.146, ~ 0) (73.8, ~ 0)	Continuum damping
(II)	Finite $\delta E_{\parallel}$, adiabatic e^- with $T_e \rightarrow T_e + 7T_i/4$, kinetic ion with $T_i \rightarrow 0.01T_i$	(0.172, -0.00615) (87.3, -3.12)	Radiative damping added on top of case (I)
(III)	Same as case (II) except for real T_e and T_i profiles	(0.168, -0.000983) (85.1, -0.499)	Ion Landau damping and pressure gradient driving added on top of case (II)
(IV)	Drift-kinetic e^- added on top of case (III)	(0.168, -0.00318) (85.1, -1.61)	e^- Landau damping added on top of case (III)
(V)	Same as case (III) except that fast ions are added in	(0.182, 0.0134) (92.6, 6.79)	Fast ion gradient driving added on top of case (III)
(VI)	Drift-kinetic e^- added on top of case (V)	(0.181, 0.0122) (92.0, 6.17)	e^- Landau damping added on top of case (V)

times the thermal velocity. For better numerical properties, the marker density and temperature are approximated to be uniform with their values taken on the profiles in figure 1 at the $q_{\min}$ surface, while the plasma gradient parameters $\kappa_{(n,T)} = -|\nabla \ln(n, T)|$ are still accurately calculated from the profiles in figure 1. This approximation can be justified by the localization of the RSAE. Collision and rotation effects are omitted in this work.

3. Damping and driving mechanisms

Kinetic simulations allow us to turn on or off various effects in each run to separate different physics. Six simulations isolating different physics of RSAE damping and driving mechanisms are performed on the $n = 3$ mode and the results are listed in table 1. Their frequencies, radial positions and mode widths are marked on the corresponding Alfvén continuum plots in figure 2. The radial mode width is defined as the full-width at half-maximum (FWHM) of the electrostatic potential $\delta\phi$ of the dominant $m = 10$ harmonic in the radial coordinate ρ . The Alfvén continua are calculated in the ideal MHD limit [37] using the poloidal-spectral method in the slow sound approximation [38] described in the appendix with an effective kinetic pressure [39] to properly account for the geodesic compressibility, i.e. the correct frequency of the geodesic acoustic mode (GAM): $\gamma_s P_0 = P_{0e} + 7P_{0i}/4$. The mode structures, including the poloidal contour plots and m -harmonic radial profiles of $\delta\phi$, of all the six cases are shown in figure 3.

In the first four cases, the fast ions are not loaded in simulations, and the thermal ion density is loaded to be the same as the electron density so as to retain charge neutrality. The mode is excited by an initial perturbation, which has an $n = 3$ and $m = 10$ harmonic and a Gaussian radial profile that peaks at $q_{\min}$. The eigenmode is measured after a few oscillations when the mode structure does not change with time anymore. Case (I) corresponds to a reduced ideal MHD model [26] with zero β . No kinetic particles are loaded and therefore there are no kinetic effects in this case. The simulation result shows that the RSAE exists in this case even with the equilibrium current, due to toroidal coupling and other

geometric effects. Due to the absence of kinetic effects, the mode is only subjected to continuum damping [32, 40], which is very small because the mode does not touch the Alfvén continuum. In case (II), finite electron temperature T_e , parallel electric field $\delta E_{\parallel}$ and kinetic ions are added on top of case (I). The electrons are simulated by the continuity equation (28) in [26] with the adiabatic pressure closure (19) and (20) in [18]. The ion pressure is artificially moved to the electrons by adding $7T_i/4$ to the electron temperature and reducing the ion temperature to 1% of its original value, in order to remove all wave-particle resonances. The frequency is elevated due to the geodesic compressibility associated with the thermal plasma pressure. Damping is enhanced due to radiative damping [33] associated with mode conversion to kinetic Alfvén wave. Physically the process of radiative damping produces small-scale structures that are eventually dissipated by effects such as the finite Larmor radius (FLR) effect, etc. Our measurement of the radiative damping rate is on a short time scale which is insensitive to the detailed dissipation physics on a long time scale. The radiative damping in case (II) can be seen in the exponential decay of the field energy in figure 4. The structure of this damped mode varies slightly over time in the initial value simulation. In case (III), real electron and ion temperatures are recovered. As a result, ion kinetic damping and pressure gradient driving are introduced. The damping rate in case (III) is smaller than that one in case (II), indicating that the ion pressure gradient driving is stronger than the ion Landau damping and FLR damping. The frequency closeness between cases (II) and (III) again confirms the calculation of the effective kinetic pressure of geodesic compressibility in [39]. In case (IV), kinetic electrons are added on top of case (III) by solving the electron perturbed pressure and the effective potential to higher order using the PIC method, as described in section III.B in [18]. Our convergence test shows that the results solved up to the first, second and third order of the fluid-kinetic hybrid electron model [41] are almost identical, so in production runs, results are solved up to the first order. Since kinetic electrons are added, electron Landau damping is introduced, enhancing the damping rate. From figure 3(e)-(h), it can be seen that the electron kinetic effects have little impact on the mode structure.

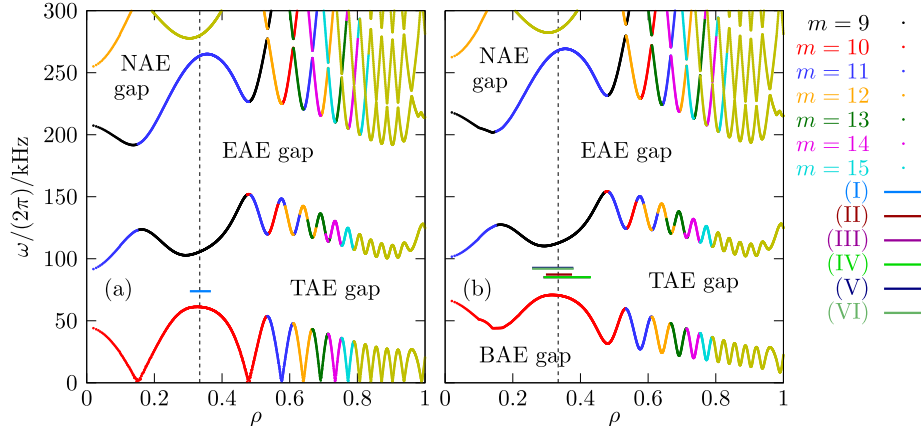


Figure 2. Alfvén continua with slow sound approximation for $n = 3$ and $q_{\min} = 3.18$: (a) zero- β limit, (b) finite- β case. The horizontal lines are the frequencies obtained in various simulation cases described in table 1. The position of each horizontal line represents the position of the mode and its width represents the FWHM of the radial mode width of $\delta\phi$ of the dominant $m = 10$ harmonic. Note that cases (III) and (IV) have almost identical frequencies and mode widths, so they are hard to distinguish in (b). Similarly, case (V) and case (VI) are also hard to distinguish. Simulations do not assume slow sound approximation. Dashed lines indicate the $q_{\min}$ position.

In case (V), fast ions are added on top of case (III). Now the poloidal mode structure is distorted as can be seen by comparing figures 3(e) and 3(i). This distortion is due to the non-perturbative contributions from the fast ions [19, 20]: the structure in figure 3(i) is more ballooning associated with the bad curvature of the magnetic field line. Stronger toroidal coupling is caused by frequency being closer to the $m = 9$ continuum and thus a larger amplitude of the sub-dominant $m = 9$ harmonic. The up-down symmetry is broken by radial symmetry breaking mainly due to the radial variations of the fast ion density gradient. In case (VI), kinetic electrons are added on top of case (V), decreasing the growth rate. Electron kinetic effects have little impact on the mode structure in this fast ion excitation case, as can be seen from figures 3(i)–(l). It can be seen from table 1 that the electron kinetic damping is different between the stable case (cases (III) and (IV)) and the unstable case (cases (V) and (VI)). This is mostly due to the mode structure difference between the stable and unstable cases and, to a lesser extent, a small difference in the real frequency. Therefore, accurate damping and growth rate calculation requires a non-perturbative and fully self-consistent simulation to calculate the true mode structure.

In the m -harmonic decomposition plots in figure 3, it can be seen that besides the dominant $m = 10$ harmonic, there is a sub-dominant $m = 9$ harmonic for cases (II)–(VI). In addition to the obvious ballooning-like mode structure, this is also partially because this particular case is during the mode transition from RSAE to TAE [8]. The mode seen in the simulation is something between an RSAE and a TAE. In one situation it may be more RSAE-like, such as case (I). In another situation it could be more TAE-like, such as cases (II)–(VI). In the next section this will be discussed in detail.

4. RSAE frequency up-sweeping and RSAE to TAE transition

To see the RSAE to TAE transition more clearly, we scan the $q_{\min}$ value based on the most comprehensive physics case (VI). Modification of the $q_{\min}$ value is carried out by adding

a constant to the q -profile. As the change in $q_{\min}$ is small ($<4\%$), the inconsistency in the equilibrium introduced by such a modification is negligible. For the $q_{\min}$ range we scan, the corresponding time range in experiment is about 58 ms. Changes of plasma profiles in the experiment are small within this time, so the plasma profiles are kept the same in simulations for the $q_{\min}$ scan.

The frequency, growth rate and radial mode width versus $q_{\min}$ are plotted in figure 5. Since the plasma rotation effect is omitted in the simulations, the experimental frequencies shown in figure 5(a) are the plasma frame frequencies calculated by subtracting the Doppler shift frequency due to plasma rotation from the frequencies measured in the lab frame. The experimental plasma rotation is 2.6 ± 0.3 kHz, so for the $n = 3$ mode, the Doppler shift is about 7.8 kHz [34]. It can be seen from figure 5(a) that the simulated mode frequencies agree well with the experimental data in both the RSAE up-sweeping region and the RSAE to TAE transition region. From figure 5(b) it can be seen that the growth rate decreases as $q_{\min}$ drops, which is consistent with the experiment in which the RSAE signal gets weaker as $q_{\min}$ drops. During the RSAE up-sweeping, the radial mode width remains roughly constant, except at the beginning of the up-sweeping where the mode is wider. At the end of the up-sweeping, the mode also gets wider because it starts to make the transition to two TAEs. The growth rates in the experiment are estimated by measuring the mode spectral width and are compared with the simulation results in figure 5(c). Note that the growth rate estimates have large uncertainties, and the fast ion distribution in our simulations could be quite different from the experiment. Nevertheless, the growth rates in simulation are quite close to the experimental ones in magnitude and in $q_{\min}$ dependence.

The Alfvén continua and mode frequencies and structures for various $q_{\min}$ values during the RSAE to TAE transition are shown in figure 6. It can be seen that as $q_{\min}$ drops, the extrema of the $m = 9$ and $m = 10$ continua move towards the global TAE gap and the sub-dominant $m = 10$ harmonic gets larger in amplitude compared with the dominant $m = 9$ harmonic. As $q_{\min}$ continues to drop, two TAE gaps of $m = 9, 10$ are created on both sides of the $q_{\min}$ surface, and then move away

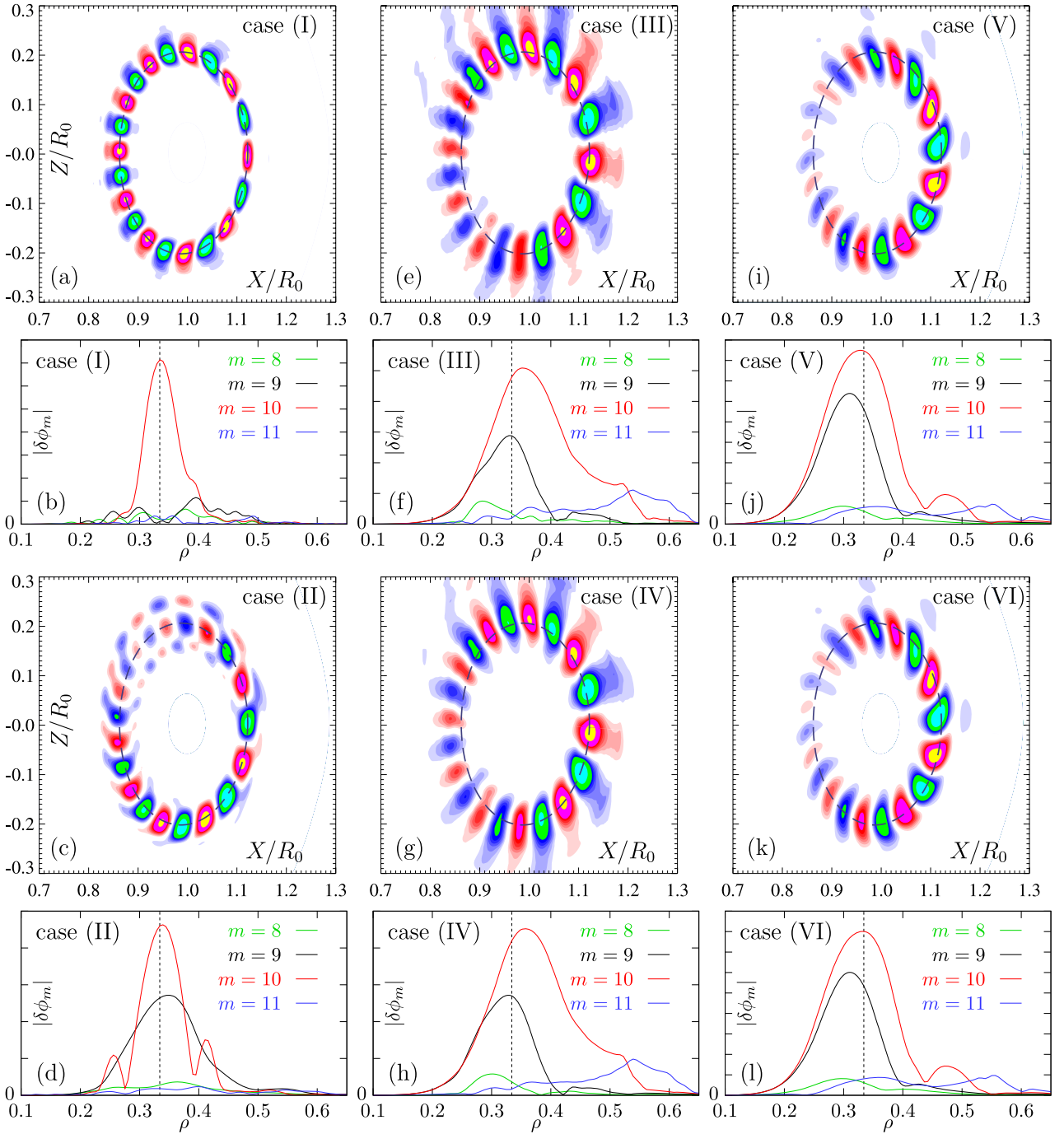


Figure 3. Poloidal contour plots and m -harmonic radial profile plots of $\delta\phi$ for the $n = 3$ mode with $q_{\min} = 3.18$ for the six cases listed in table 1. Dashed lines indicate the $q_{\min}$ position.

from the $q_{\min}$ surface. As a result, the original RSAE breaks up into two TAEs.

5. Equilibrium magnetic field and current direction

It is straightforward to show that the ideal MHD eigenmode equation (53) in [26] in the cold plasma limit is invariant in an up-down symmetric geometry while changing the equilibrium toroidal magnetic field or toroidal current directions. In our simulations with kinetic effects and realistic geometry, changing the toroidal field or current direction would cause

only about 0.1% difference in mode frequency and 2% difference in growth rate, which are negligible considering the measurement error and the particle statistical uncertainty. As the up-down asymmetry is weak in this case, the mode structure simply becomes upside down when switching the toroidal field direction, while it is unchanged when switching the current direction, as shown in figure 7. Such conclusions are also drawn in GYRO and TAEFL simulations [34]. Moreover, the mode frequencies, growth rates and mode structures in GYRO and TAEFL simulations are also close to the GTC results [34].

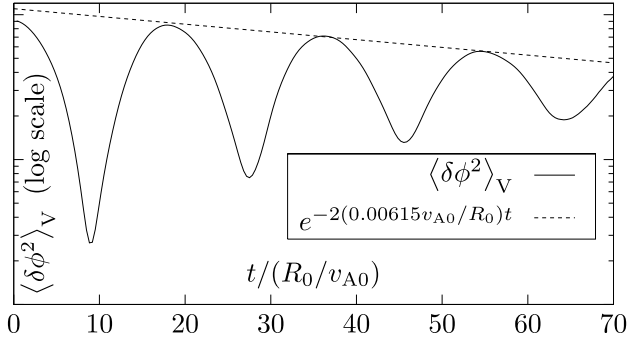


Figure 4. Exponential decay of volume-averaged $\delta\phi^2$ in time for case (II) shows the radiative damping even in the $T_i \rightarrow 0$ limit.

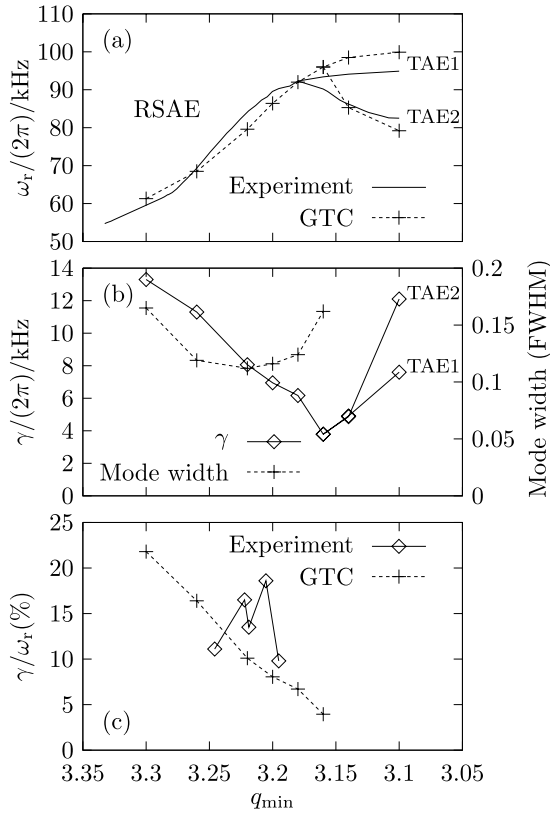


Figure 5. RSAE/TAE mode frequency, growth rate and radial mode width dependence on $q_{\min}$: (a) frequency comparison between simulation and experiment; experimental values are obtained by reading the experimental spectrogram provided by M Van Zeeland [34], and have subtracted Doppler shift frequency (7.8 kHz) due to plasma rotation; (b) growth rate and radial mode width (FWHM in ρ) from simulation; (c) growth rate comparison between simulation and experiment; experimental growth rate data are provided by B Tobias.

6. Summary

Linear properties of the reverse shear Alfvén eigenmode (RSAE) in a well-diagnosed DIII-D tokamak experiment (discharge #142111) are studied in gyrokinetic particle simulations. Simulations find that a weakly damped RSAE exists due to toroidal coupling and other geometric effects. The mode is driven unstable by density gradients of fast ions from neutral beam injection. Various damping and driving mechanisms are identified and measured in the simulations.

Accurate damping and growth rate calculation requires a non-perturbative and fully self-consistent simulation to calculate the true mode structure. The mode structure has no up-down symmetry mainly due to the radial symmetry breaking by the density gradients of the fast ions, as measured in the experiment by electron cyclotron emission imaging. The RSAE frequency up-sweeping and the mode transition from RSAE to TAE (toroidal Alfvén eigenmode) are in good agreement with the experimental results when the values of the minimum safety factor are scanned in gyrokinetic simulations.

Acknowledgments

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Appendix. A poloidal-spectral method for numerical calculation of Alfvén continuum

In realistic situations, simple estimation of the Alfvén continuum like $\omega_A \approx (nq - m)v_A/(qR_0)$ is not good enough. Such an estimation would introduce fairly large inaccuracy by geometric effects, finite- β effect, etc. In this section a poloidal-spectral method is used to numerically solve the ideal MHD Alfvén continuum equation [37], which writes

$$\mathbb{E} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = \begin{pmatrix} \mathbb{E}_{11} & \mathbb{E}_{12} \\ \mathbb{E}_{21} & \mathbb{E}_{22} \end{pmatrix} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = 0, \quad (\text{A.1})$$

where

$$\mathbb{E}_{11} = \frac{4\pi\rho_M\omega^2|\nabla\psi|^2}{B_0^2} + B_0 \cdot \nabla \left(\frac{|\nabla\psi|^2 B_0 \cdot \nabla}{B_0^2} \right), \quad (\text{A.2})$$

$$\mathbb{E}_{12} = 4\pi\gamma_s P_0 \kappa_s, \quad (\text{A.3})$$

$$\mathbb{E}_{21} = \kappa_s, \quad (\text{A.4})$$

$$\mathbb{E}_{22} = \frac{4\pi\gamma_s P_0 + B_0^2}{B_0^2} + \frac{\gamma_s P_0}{\rho_M \omega^2} B_0 \cdot \nabla \left(\frac{B_0 \cdot \nabla}{B_0^2} \right), \quad (\text{A.5})$$

$$\kappa = b_0 \cdot \nabla b_0 = (\nabla \times b_0) \times b_0, \quad (\text{A.6})$$

$$\kappa_s = 2\kappa \cdot \frac{B_0 \times \nabla\psi}{B_0^2}. \quad (\text{A.7})$$

The equilibrium magnetic field in magnetic coordinates [18, 42] reads

$$B_0 = g(\psi)\nabla\zeta + I(\psi)\nabla\theta + \delta(\psi, \theta)\nabla\psi \quad (\text{A.8})$$

$$= q\nabla\psi \times \nabla\theta - \nabla\psi \times \nabla\zeta. \quad (\text{A.9})$$

Then some vector expressions can be simplified and become scalar ones:

$$B_0 \cdot \nabla = \mathcal{J}^{-1}(\partial_\theta + q\partial_\zeta), \quad (\text{A.10})$$

$$\begin{aligned} \nabla \times b_0 = & \left[\partial_\psi \left(\frac{I}{B_0} \right) - \partial_\theta \left(\frac{\delta}{B_0} \right) \right] \nabla\psi \times \nabla\theta \\ & + \partial_\psi \left(\frac{g}{B_0} \right) \nabla\psi \times \nabla\zeta + g \left(\frac{\partial_\theta}{B_0} \right) \nabla\theta \times \nabla\zeta, \end{aligned} \quad (\text{A.11})$$

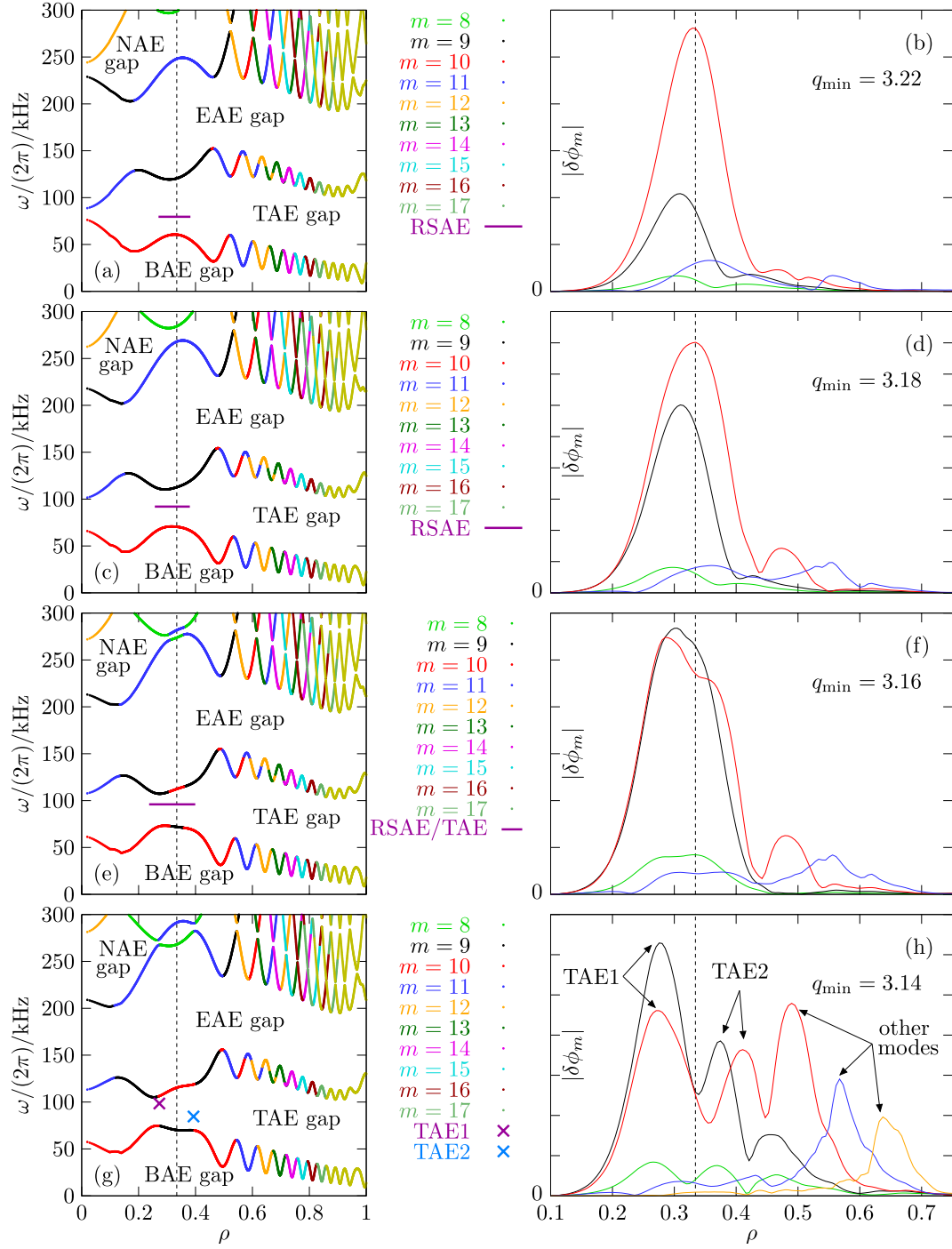


Figure 6. RSAE to TAE transition in GTC simulations. In the left column are the Alfvén continua and the mode frequencies. In the right column are the m -harmonic radial profiles of $\delta\phi$. (a), (b) $q_{\min} = 3.22$; (c), (d) $q_{\min} = 3.18$; (e), (f) $q_{\min} = 3.16$. (g), (h) $q_{\min} = 3.14$. Dashed lines indicate the $q_{\min}$ position.

$$\kappa = \frac{\mathcal{J}^{-1}}{B_0} \left\{ \left[\partial_\theta \left(\frac{\delta}{B_0} \right) - \partial_\psi \left(\frac{I}{B_0} \right) - q \partial_\psi \left(\frac{g}{B_0} \right) \right] \nabla \psi - g \left(\partial_\theta \frac{1}{B_0} \right) q \nabla \theta + g \left(\partial_\theta \frac{1}{B_0} \right) \nabla \zeta \right\}, \quad (\text{A.12})$$

$$\kappa_s = -\frac{2\mathcal{J}^{-1}}{B_0} g \left(\partial_\theta \frac{1}{B_0} \right) = \frac{2\mathcal{J}^{-1}g}{B_0^3} \partial_\theta B_0.$$

In GTC, $|\nabla\psi|^2$ can be calculated using the spline functions of the cylindrical coordinates (X, Z, ζ) :

$$|\nabla\psi|^2 = (\partial_X \psi)^2 + (\partial_Z \psi)^2 = \left(\frac{1}{\partial_\psi X - \partial_\theta X \frac{\partial_\psi Z}{\partial_\theta Z}} \right)^2 + \left(\frac{1}{\partial_\psi Z - \partial_\theta Z \frac{\partial_\psi X}{\partial_\theta X}} \right)^2. \quad (\text{A.14})$$

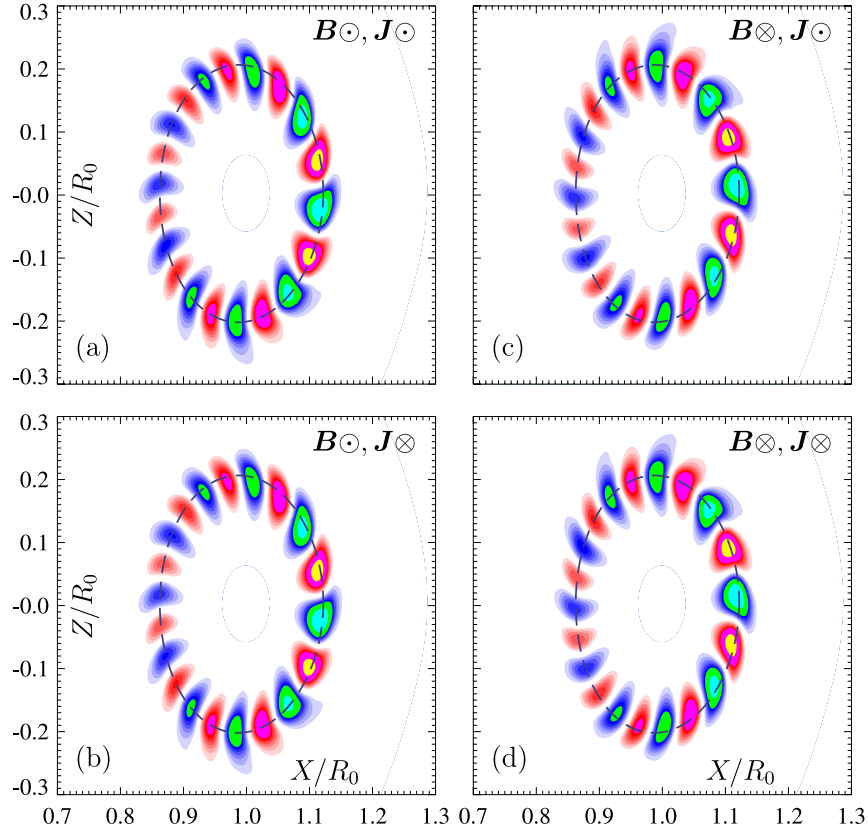


Figure 7. Mode structure of $q_{\min} = 3.22$ dependence on the equilibrium toroidal magnetic field and current directions.

For convenience, scale each equation in (A.1) by a coefficient: where

$$\begin{pmatrix} 1/\mathcal{J}^{-1} & 0 \\ 0 & 4\pi\varrho_M\omega^2/\mathcal{J}^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{E}_{11} & \mathbb{E}_{12} \\ \mathbb{E}_{21} & \mathbb{E}_{22} \end{pmatrix} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = \begin{pmatrix} \mathbb{E}_{11}/\mathcal{J}^{-1} & \mathbb{E}_{12}/\mathcal{J}^{-1} \\ 4\pi\varrho_M\omega^2\mathbb{E}_{21}/\mathcal{J}^{-1} & 4\pi\varrho_M\omega^2\mathbb{E}_{22}/\mathcal{J}^{-1} \end{pmatrix} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = 0. \quad (\text{A.15})$$

$$\begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = \begin{pmatrix} \vdots \\ (\xi_s)_{m-1} \\ (\xi_s)_m \\ (\xi_s)_{m+1} \\ \vdots \\ (\nabla \cdot \xi)_{m-1} \\ (\nabla \cdot \xi)_m \\ (\nabla \cdot \xi)_{m+1} \\ \vdots \end{pmatrix}, \quad (\text{A.19})$$

This equation can be solved in the poloidal Fourier space by expanding the θ -dependent quantities:

$$\begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = e^{in\zeta} \sum_m \begin{pmatrix} (\xi_s)_m \\ (\nabla \cdot \xi)_m \end{pmatrix} e^{-im\theta}, \quad (\text{A.16})$$

$$\mathbb{A} = \begin{pmatrix} \mathbb{G}^\dagger \mathbb{H} \mathbb{G} & -4\pi\gamma_s P_0 \mathbb{K} \\ 0 & 4\pi\gamma_s P_0 \mathbb{G}^\dagger \mathbb{M} \mathbb{G} \end{pmatrix}, \quad (\text{A.20})$$

$$\mathbb{B} = \begin{pmatrix} \mathbb{J} & 0 \\ \mathbb{K} & \mathbb{L} \end{pmatrix}. \quad (\text{A.21})$$

$$\begin{pmatrix} \frac{|\nabla\psi|^2 \mathcal{J}^{-1}}{B_0^2} \\ \frac{|\nabla\psi|^2}{B_0^2 \mathcal{J}^{-1}} \\ \frac{\kappa_s}{\mathcal{J}^{-1}} \\ \frac{4\pi\gamma_s P_0 + B_0^2}{B_0^2 \mathcal{J}^{-1}} \end{pmatrix} = \sum_m \begin{pmatrix} \left(\frac{|\nabla\psi|^2 \mathcal{J}^{-1}}{B_0^2} \right)_m \\ \left(\frac{|\nabla\psi|^2}{B_0^2 \mathcal{J}^{-1}} \right)_m \\ \left(\frac{\kappa_s}{\mathcal{J}^{-1}} \right)_m \\ \left(\frac{4\pi\gamma_s P_0 + B_0^2}{B_0^2 \mathcal{J}^{-1}} \right)_m \end{pmatrix} e^{im\theta}. \quad (\text{A.17})$$

Using $\{e^{-im\theta}\}$ as the basis, (A.15) can be written in this matrix form:

$$\mathbb{A} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = 4\pi\varrho_M\omega^2 \mathbb{B} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix}, \quad (\text{A.18})$$

The operator matrices and their elements are

$$\mathbb{G} = -i \frac{\mathbf{B}_0 \cdot \nabla}{\mathcal{J}^{-1}}, \quad \mathbb{G}_{m,m'} = (nq - m)\delta_{m,m'}, \quad (\text{A.22})$$

$$\mathbb{H} = \frac{|\nabla\psi|^2 \mathcal{J}^{-1}}{B_0^2}, \quad \mathbb{H}_{m,m'} = \left(\frac{|\nabla\psi|^2 \mathcal{J}^{-1}}{B_0^2} \right)_{m'-m}, \quad (\text{A.23})$$

$$\mathbb{J} = \frac{|\nabla\psi|^2}{B_0^2 \mathcal{J}^{-1}}, \quad \mathbb{J}_{m,m'} = \left(\frac{|\nabla\psi|^2}{B_0^2 \mathcal{J}^{-1}} \right)_{m'-m}, \quad (\text{A.24})$$

$$\mathbb{K} = \frac{\kappa_s}{\mathcal{J}^{-1}}, \quad \mathbb{K}_{m,m'} = \left(\frac{\kappa_s}{\mathcal{J}^{-1}} \right)_{m'-m}, \quad (\text{A.25})$$

$$\mathbb{L} = \frac{4\pi\gamma_s P_0 + B_0^2}{B_0^2 \mathcal{J}^{-1}}, \quad \mathbb{L}_{m,m'} = \left(\frac{4\pi\gamma_s P_0 + B_0^2}{B_0^2 \mathcal{J}^{-1}} \right)_{m'-m}, \quad (\text{A.26})$$

$$\mathbb{M} = \frac{\mathcal{J}^{-1}}{B_0^2} = \frac{1}{gq + I} \mathbb{I}, \quad \mathbb{M}_{m,m'} = \frac{1}{gq + I} \delta_{m,m'}. \quad (\text{A.27})$$

Using the normalization in GTC [18], the matrices are normalized as

$$\hat{\mathbb{H}} = \frac{\mathbb{H}}{B_a R_0}, \quad (\text{A.28})$$

$$\hat{\mathbb{J}} = \frac{B_a}{R_0^3} \mathbb{J}, \quad (\text{A.29})$$

$$\hat{\mathbb{K}} = \frac{B_a}{R_0} \mathbb{K}, \quad (\text{A.30})$$

$$\hat{\mathbb{L}} = \frac{B_a}{R_0} \mathbb{L}, \quad (\text{A.31})$$

$$\hat{\mathbb{M}} = B_a R_0 \mathbb{M}, \quad (\text{A.32})$$

$$\hat{\mathbb{A}} = \begin{pmatrix} 1/(B_a R_0) \mathbb{I} & 0 \\ 0 & (R_0/B_a) \mathbb{I} \end{pmatrix} \mathbb{A}, \quad (\text{A.33})$$

$$\hat{\mathbb{B}} = \begin{pmatrix} (B_a/R_0^3) \mathbb{I} & 0 \\ 0 & (B_a/R_0) \mathbb{I} \end{pmatrix} \mathbb{B}. \quad (\text{A.34})$$

Normalize ϱ_M and ω as

$$\hat{\varrho}_M = \frac{\varrho_M}{n_a m_p}, \quad (\text{A.35})$$

$$\hat{\omega} = \frac{R_0}{v_{Ap}} \omega, \quad (\text{A.36})$$

where

$$v_{Ap} \equiv \frac{B_a}{\sqrt{4\pi n_a m_p}}. \quad (\text{A.37})$$

Then (A.18) is normalized as

$$\hat{\mathbb{A}} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix} = \hat{\varrho}_M \hat{\omega}^2 \hat{\mathbb{B}} \begin{pmatrix} \xi_s \\ \nabla \cdot \xi \end{pmatrix}, \quad (\text{A.38})$$

which is a generalized eigenvalue problem. A code named ALCON based on the eigenvalue library SLEPc [43] is developed to solve this equation using the equilibrium data in GTC. The solution for the simulation scenario in section 3 is shown in figure A1.

Since the sound continua are not a major concern for RSAE and they make the continua plot very messy, they can be removed using the slow sound approximation [38]. Comparing the second term of $\mathbb{E}_{22}$ with the first term gives

$$\begin{aligned} \frac{\gamma_s P_0}{\varrho_M \omega^2} \mathbf{B}_0 \cdot \nabla \left(\frac{\mathbf{B}_0 \cdot \nabla}{B_0^2} \right) &\approx \frac{-4\pi\gamma_s P_0 k_{\parallel}^2 / (4\pi\varrho_M \omega^2)}{(4\pi\gamma_s P_0 + B_0^2)/B_0^2} \\ &\approx \frac{-4\pi\gamma_s P_0 / B_0^2}{(4\pi\gamma_s P_0 + B_0^2)/B_0^2} \sim O\left(\frac{\beta}{\beta + 1}\right), \end{aligned} \quad (\text{A.39})$$

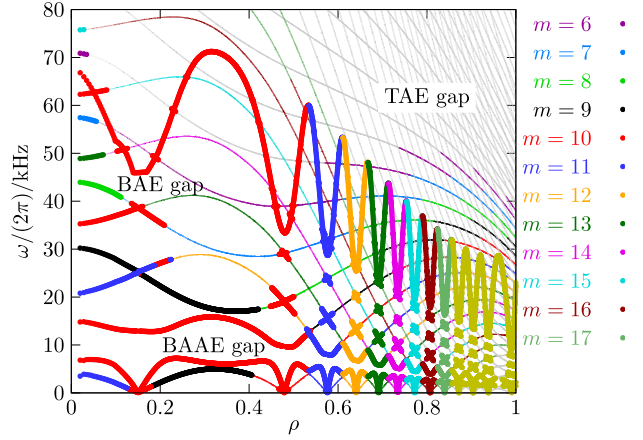


Figure A1. Alfvén and sound continua solved for $n = 3$, $m \in [-20, 47]$ of DIII-D #142111 at 745 ms. The thick lines are the Alfvén branches and the thin ones are the sound branches.

which means the second term of $\mathbb{E}_{22}$ can be dropped in the $\beta \ll 1$ limit. In this slow sound approximation, (A.1) becomes

$$\begin{aligned} &\left[4\pi\varrho_M \omega^2 \frac{|\nabla\psi|^2}{\mathcal{J}^{-1} B_0^2} + \frac{\mathbf{B}_0 \cdot \nabla}{\mathcal{J}^{-1}} \left(\frac{|\nabla\psi|^2 \mathbf{B}_0 \cdot \nabla}{B_0^2} \right) \right. \\ &\quad \left. - \frac{4\pi\gamma_s P_0 \kappa_s^2 B_0^2}{\mathcal{J}^{-1} (4\pi\gamma_s P_0 + B_0^2)} \right] \xi_s = 0, \end{aligned} \quad (\text{A.40})$$

or in the normalized matrix form:

$$(\mathbb{G}^\dagger \hat{\mathbb{H}} \mathbb{G} + \hat{\mathbb{N}}) \xi_s = \hat{\varrho}_M \hat{\omega}^2 \hat{\mathbb{J}} \xi_s, \quad (\text{A.41})$$

where the operator matrix $\mathbb{N}$, its elements and its normalization are given by

$$\mathbb{N} = \frac{4\pi\gamma_s P_0 \kappa_s^2 B_0^2}{\mathcal{J}^{-1} (4\pi\gamma_s P_0 + B_0^2)}, \quad (\text{A.42})$$

$$\mathbb{N}_{m,m'} = \left(\frac{4\pi\gamma_s P_0 \kappa_s^2 B_0^2}{\mathcal{J}^{-1} (4\pi\gamma_s P_0 + B_0^2)} \right)_{m'-m}, \quad (\text{A.43})$$

$$\hat{\mathbb{N}} = \frac{\mathbb{N}}{B_a R_0}. \quad (\text{A.44})$$

By comparing the continua solved in the slow sound approximation in figure 2(b) and the accurate solution in figure A1, we find that the slow sound approximation is good enough to present the Alfvén continua in this simulation scenario. As a result, to avoid the distraction by the sound continua, all Alfvén continua shown in sections 3 and 4 are in the slow sound approximation.

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